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TO'G'RI TO'RTBURCHAKDA IKKITA ICHKI TIP O'ZGARISH CHIZIG'IGA EGA BO'LGAN ARALASH TIPDAGI TENGLAMA UCHUN DIRIXLE MASALASI

ЗАДАЧА ДИРИХЛЕ ДЛЯ УРАВНЕНИЯ СМЕШАННОГО ТИПА С ДВУМЯ ЛИНИЯМИ ПЕРЕХОДА В ПРЯМОУГОЛЬНОЙ ОБЛАСТИ

THE DIRICHLET PROBLEM FOR A MIXED-TYPE EQUATION WITH TWO VARIABLE BOUNDARY CONDITIONS IN A RECTANGLE

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Annotatsiya

Ikkita ichki perpendikulyar tip o'zgarish chizig'iga ega bo'lgan elliptik-giperbolik tipdagi tenglama uchun to'g'ri to'rtburchakda birinchi chegaraviy masala o'rganilgan. Masala yechimining yagonaligi va mavjudligi isbotlangan. Yechimning yagonaligini isbotlashda masalaning xos funksiyalari sistemasining L_2 fazoda to'laligidan foydalanilgan. Masalaning yechimini esa qator ko'rinishida qurilgan va kichik maxrajlar muammosi paydo bo'lgan. Maxrajning noldan farqli ekanligi ko'rsatilgan.

Аннотация

Изучена первая краевая задача для уравнения эллиптико-гиперболического типа с двумя перпендикулярными внутренними линиями изменения типа и спектральным параметром. Доказаны единственность и существование решения. При доказательстве единственности используется полнота в пространстве L_2 системы, биортогонально сопряженной с системой собственных функций соответствующей одномерной задачи. При построении решения в виде суммы ряда по биортогональной системе функций возникает проблема малых знаменателей. Получены оценки об отделимости знаменателей от нуля.

Abstract

The first boundary value problem for an elliptic-hyperbolic type equation with two internal perpendicular type change lines has been studied in a rectangular domain. The existence and uniqueness of the solution to the problem have been proven. The uniqueness of the solution was established using the completeness of the system of eigenfunctions in the corresponding space. The solution to the problem was constructed in the form of a series, leading to the issue of small denominators. It was shown that the denominator is nonzero.

Kalit so'zlar: *aralash tipdagi tenglama, Dirixle masalasi, yechimning mavjudligi va yagonaligi, spektral usul.*

Ключевые слова: *уравнение смешанного типа, задача Дирихле, единственность, существование решения, спектральный метод.*

Key words: *mixed-type differential equation, Dirichle problem, existence and unique of the problem, spectral method.*

KIRISH

Ichki tip o'zgarish chizig'iga ega bo'lgan aralash tipdagi tenglamalar uchun chegaraviy masalar ko'plab tadqiqotchilarning e'tiborini tortgan va hozirgi kunda ham bu mavzu qiziqarli va dolzarb mavzulardan biri bo'lib qolmoqda. Bu mavzu bo'yicha dunyoning turli burchaklarida ko'plab tadqiqotchilar ish olib bormoqdalar. Bu tadqiqotlarda to'g'ri to'rtburchakda ichki tip o'zgarish chizig'iga ega bo'lgan aralash tipdagi turli xil tenglamalar uchun Dirixle masalasi o'rganilgan.

ADABIYOTLAR TAHLILI VA METODOLOGIYA

$[2]$ ishda Trikomi masalasi klassik aralash tipdagi tenglamalar uchun yechilgan bo‘lib, unda giperbolik qism tenglamaning xarakteristikalari va tip o‘zgarish chiziqlari bilan chegaralangan ikkita sohadan iborat. Bu masalaning yetarlicha sharhlari mavjud.

Ikkita ichki tip o‘zgarish chizig‘iga ega tenglama uchun chegaraviy masala $[3]$ ishda o‘rganilgan. Birinchi qism to‘rtta elliptik qismdan va to‘rtta giperbolik qismdan, ikkinchi qism esa berilgan tenglamaning xarakteristikalari va tip o‘zgarish chiziqlari bilan chegaralangan.

Tranzonik gazga oid ba’zi masalalar $[4]$ ishda ko‘rsatilgan. Bu masalar aralash tipdagi tenglamalar uchun Dirixle masalasiga keltiriladi.

$[5]$ ishda Lavrentev tenglamasi uchun Dirixle masalasining notog‘ri qo‘yilganligi isbotlangan. Keyinchalik aralash tipdagi tenglamalar uchun Dirixle masalasi ko‘plab tadqiqotchilarning e’tiborini tortgan, jumladan $[6-11]$ ishlarni sanab o‘tish mumkin. Bu ishlarda ichki tip o‘zgarish chizig‘iga ega bo‘lgan aralash tipdagi tenglamalar uchun Dirixle masalasini yechishning yagonaligi ekstremum prinsipi va integral ayniyatlar usuli yordamida, uning mavjudligi esa o‘zgaruvchilarni ajratish usuli yordamida isbotlangan.

$[12,13]$ ishda esa to‘rtburchak soha chegarasida ichki tip o‘zgarish chizig‘iga ega bo‘lgan aralash tipdagi tenglama uchun Dirixle masalasi o‘rganilgan. Spektrlar analiz usuli yordamida yagonalik mezoni o‘rnatilgan va masalaning yechimi xos funksiyalar sistemasining yig‘indisidan iborat qator ko‘rinishida qurilgan.

Spektral analiz usuli yordamida singulyar koeffitsentli aralash tipdagi tenglamalar uchun chegaraviy masalalar bilan A.K.O‘rinov va K.T.Kamoliddinovlar $[17]$,$[18]$ ishlarda shug‘ullanishgan.

Mazkur ishda spektral parametrga ega bo‘lgan ikkita ichki perpendikular tip o‘zgarish chizig‘iga ega bo‘lgan tenglama uchun to‘rtburchak sohada Dirixle masalasi uchun spektral masalaning xos qiymatlari topilgan. Yechimning yagonalik shartlari qo‘yilgan va (2)-(5)masalaning yechimi qator ko‘rinishida qurilgan. Ilgari bu g‘oya giperbolik tipdagi tenglamalar uchun boshlang‘ich chegaraviy masala yechimining yagonaligini isbotlash uchun $[14]$ va bir aralash tipdagi tenglama uchun $[12]$ ishlarda qo‘llanilgan. (2)-(5) masala yechimining mavjudligini isbotlashda, xuddi $[15,16,12]$ kabi “ kichik maxrajlar muammosi” deb ataladigan muammo yuzaga keladi, bu esa (2) funksiyalar sinfida tuzilgan qatorlarning yaqinlashuvchiligini isbotlashda qiyinchilik tug‘diradi. Masalaning yechimini beruvchi funksiyaning maxraji noldan farqliligi haqidagi lemmalar isbotlangan.

NATIJA VA MUHOKAMA

1. Masalaning qo‘yilishi. Quyidagi

$$Lu \equiv sgnx \cdot u_{xx} + \operatorname{sgn} y \cdot u_{yy} + \lambda u = 0, \tag{1}$$

tenglamani

$$D = \left\{ (x, y) \in R^2, -1 < x < 1, -\alpha < y < \beta \right\},$$

sohada qaraylik, bu yerda $\lambda \in C,\ \alpha, \beta \in R;\ \alpha, \beta > 0.$ Quyidagi belgilashlarni kiritamiz:

$$D_1 = D \cap \{x > 0, y > 0\}, \quad D_2 = D \cap \{x > 0, y < 0\},$$

$$D_3 = D \cap \{x < 0, y < 0\}, \ D_4 = D \cap \{x < 0, y > 0\}.$$

D sohada (1) tenglama uchun quyidagi masalani tadqiq qilamiz:

Dirixle masalasi. Quyidagi

$$u(x, y) \in C(\overline{D}) \cap C^1(D) \cap C^2(D_1 \cup D_2 \cup D_3 \cup D_4), \tag{2}$$

$$Lu(x, y) \equiv 0,\ (x, y) \in D_1 \cup D_2 \cup D_3 \cup D_4, \tag{3}$$

$$u(x,y)|_{x=1}=u(x,y)|_{x=-1}=0,\ y\in[-\alpha,\beta], \tag{4}$$

$$u(x,y)|_{y=-\alpha}=\psi(x),\ \ u(x,y)|_{y=\beta}=\varphi(x),\ x\in[-1;1], \tag{5}$$

shartlarni qanoatlantiruvchi $u(x,y)$ funksiya topilsin, bu yerda φ va ψ - berilgan yetarlicha silliq funksiyalar boʻlib, $\varphi(\pm1)=\psi(\pm1)=0$ kelishuv sharti oʻrinli.

1. Spektral masala. Biortogonal sistema qurish.

D **masala.** λ parametrning shunday qiymatlari topilsinki, unga mos keladigan $u(x,y)$ funskiya (2)-(5) shartlarni qanoatlantirsin. Bu yerda $\varphi(x)=\psi(x)\equiv 0$. Oʻzgaruvchilarni $u(x,y)=X(x)\cdot\ Y(y)$ koʻrinishda ajratgandan soʻng ikkita differensial tenglamaga ega boʻlamiz:

$$signx\cdot\ X''+d\cdot\ X=0,\qquad x\in(-1,0)\cup(0,1), \tag{6}$$

$$signy\cdot\ Y''+(\lambda-d)Y=0,\qquad y\in(-\alpha,0)\cup(0,\beta), \tag{7}$$

bu yerda $d=\mu^2$ ajralish oʻzgarmasi va masalaning shartlariga koʻra quyidagilar bajarilishi kerak:

$$X(0-0)=X(0+0),\quad X'(0-0)=X'(0+0), \tag{8}$$

$$X(-1)=X(1)=0\ , \tag{9}$$

$$Y(0-0)=Y(0-0),\quad Y'(0-0)=Y'(0+0), \tag{10}$$

$$Y(-\alpha)=Y(\beta)=0 \tag{11}$$

$\{(6),(8),(9)\}$ va $\{(7),(10),(11)\}$ spektral masalalar klassik masalalar hisoblanmaydi. (8) shartni qanoatlantiruvchi (6) tenglamaning yechimi

$$X(x)=\begin{cases}C_1\cos\mu x+C_2\sin\mu x, & x>0,\\ C_2ch\mu x+C_2sh\mu x, & x<0,\end{cases}$$

koʻrinishda boʻlib bu yerda C_1,C_2 -ixtiyoriy konstantalar va (9) shartni hisobga olgan holda biz quyidagi tenglamaga kelamiz:

$$tg\mu=-th\mu. \tag{12}$$

(12) tenglama yechimlari mavjudligi haqidagi quyidagi lemmani isbotlaymiz.

Lemma 1. Quyidagi

$$tg(az)=-th(bz),\quad a,b\in R,\quad a,b>0,$$

tenglama nol, tub haqiqiy va sof mavhum qarama-qarshi ishorali sonlardan tashkil topgan sanoqli ildizlar toʻplamiga ega boʻlib, ular uchun quyidagi asimptotik koʻrinishlar oʻrinli:

$$z_k^{(1),(2)}=\pm(-\frac{\pi}{4a}+\frac{\pi}{a}k+O(e^{(-2\pi kb/a)}),$$

$$z_k^{(3),(4)}=\pm(-\frac{\pi}{4b}+\frac{\pi}{b}k+O(e^{(-2\pi ka/b)}),\quad k\in N.$$

(12) tenglama, 1-lemmaga asosan $\pm\mu_k$ haqiqiy va $\pm i\mu_k$ sof mavhum ildizlarga ega va ularning asimptotik koʻrinishi quyidagicha aniqlanadi:

$$\mu_k=\left(-\frac{\pi}{4}+\pi k+O(e^{(-2\pi k)}\right). \tag{13}$$

Bu maʼlumotlarning isboti II.2. boʻlimda grafik usulda koʻrsatib oʻtilgan.

d quyidagi $d_k^{(1)}=\mu_k^2>0$ yoki $d_k^{(1)}=-\mu_k^2<0$, shartlarni qanoatlantirganda $\{(6),(8),(9)\}$ masalaning yechimlari mos ravishda quyidagicha boʻladi:

$$X_k^{(1)}(x)=\begin{cases}\dfrac{\sin[\mu_k(x-1)]}{\cos\mu_k}, & x>0,\\ \dfrac{sh[\mu_k(x+1)]}{ch\mu_k}, & x<0,\end{cases}\qquad X_k^{(2)}(x)=\begin{cases}\dfrac{\sin[\mu_k(x-1)]}{\cos\mu_k}, & x>0,\\ \dfrac{sh[\mu_k(x+1)]}{ch\mu_k}, & x<0.\end{cases}$$

(10) shartni hisobga olgan holda (7) tenglamaning topilgan μ_k larga mos yechimlari quyidagicha bo'ladi:

$$Y_k^{(1)}(y)=\begin{cases}a_k^{(1)}ch(y\sqrt{\mu_k^2-\lambda})+b_k^{(1)}sh(y\sqrt{\mu_k^2-\lambda}), & y>0,\\ a_k^{(1)}\cos(y\sqrt{\mu_k^2-\lambda})+b_k^{(1)}\sin(y\sqrt{\mu_k^2-\lambda}), & y<0,\end{cases}\tag{14}$$

$$Y_k^{(2)}(y)=\begin{cases}a_k^{(2)}\cos(y\sqrt{\mu_k^2+\lambda})+b_k^{(2)}\sin(y\sqrt{\mu_k^2+\lambda}), & y>0,\\ a_k^{(2)}ch(y\sqrt{\mu_k^2+\lambda})+b_k^{(2)}sh(y\sqrt{\mu_k^2+\lambda}), & y<0.\end{cases}\tag{15}$$

So'ngra bu yerda $a_k^{(1)},b_k^{(1)},a_k^{(2)},b_k^{(2)}-$hozircha noma'lum sonlar. (14) va (15) funksiyalarni (11) chegaraviy shartlarga bo'ysundirib, $a_k^{(j)}$, $b_k^{(j)}$, $j=1,2$ noma'lum koeffitsentlarni topish uchun quyidagi sistemaga ega bo'lamiz:

$$\begin{cases}a_k^{(1)}ch(\beta\sqrt{\mu_k^2-\lambda})+b_k^{(1)}sh(\beta\sqrt{\mu_k^2-\lambda})=0,\\ a_k^{(1)}\cos(\alpha\sqrt{\mu_k^2-\lambda})-b_k^{(1)}\sin(\alpha\sqrt{\mu_k^2-\lambda})=0,\end{cases}\tag{16}$$

$$\begin{cases}a_k^{(2)}\cos(\beta\sqrt{\mu_k^2+\lambda})+b_k^{(2)}\sin(\beta\sqrt{\mu_k^2+\lambda})=0,\\ a_k^{(2)}ch(\alpha\sqrt{\mu_k^2+\lambda})-b_k^{(j)}sh(\alpha\sqrt{\mu_k^2+\lambda})=0.\end{cases}\tag{17}$$

(16) va (17) sistemalar notrivial yechimlarga ega bo'ladi, faqat va faqat barcha $k\in N$ uchun ushbu sistemalarning determinantlari 0 ga teng bo'lsa, ya'ni

$$\Delta_k^{(1)}(\alpha,\beta)=\cos\left(\alpha\sqrt{\mu_k^2-\lambda}\right)sh\left(\beta\sqrt{\mu_k^2-\lambda}\right)+\sin\left(\alpha\sqrt{\mu_k^2-\lambda}\right)ch\left(\beta\sqrt{\mu_k^2-\lambda}\right)=0,\tag{18}$$

$$\Delta_k^{(2)}(\alpha,\beta)=ch\left(\alpha\sqrt{\mu_k^2+\lambda}\right)\sin\left(\beta\sqrt{\mu_k^2+\lambda}\right)+sh\left(\alpha\sqrt{\mu_k^2+\lambda}\right)\cos\left(\beta\sqrt{\mu_k^2+\lambda}\right)=0.\tag{19}$$

ushbu holatda

$$b_k^1=a_k^{(1)}ctg\left(a\sqrt{\mu_k^2-\lambda}\right),\quad b_k^{(2)}=a_k^{(2)}cth\left(a\sqrt{\mu_k^2+\lambda}\right),$$

bo'ladi va (14) ,(15) funksiyalar quyidagi ko'rinishni oladi:

$$Y_k^{(1)}(y)=\begin{cases}a_k^{(1)}\cos\left(y\sqrt{\mu_k^2-\lambda}\right)+ctg\left(\left(\alpha\sqrt{\mu_k^2-\lambda}\right)\sin\left(y\sqrt{\mu_k^2-\lambda}\right)\right), & y<0,\\ a_k^{(1)}ch\left(\left(y\sqrt{\mu_k^2-\lambda}\right)+ctg\left(\alpha\sqrt{\mu_k^2-\lambda}\right)sh\left(y\sqrt{\mu_k^2-\lambda}\right)\right), & y>0,\end{cases}$$

$$Y_k^{(2)}(y)=\begin{cases}a_k^{(2)}\left(ch\left(y\sqrt{\mu_k^2+\lambda}\right)+cth\left(\alpha\sqrt{\mu_k^2+\lambda}\right)sh\left(y\sqrt{\mu_k^2+\lambda}\right)\right), & y<0,\\ a_k^{(2)}\left(\cos\left(y\sqrt{\mu_k^2+\lambda}\right)+th\left(\alpha\sqrt{\mu_k^2+\lambda}\right)\sin\left(y\sqrt{\mu_k^2+\lambda}\right)\right), & y>0,\end{cases}$$

bu yerda $a_k^{(1)}$, $a_k^{(2)}$ -ixtiyoriy koeffitsentlar.

2-lemma. D_λ spektral masalaning xos qiymatlari quyidagicha aniqlanadi:

$$\lambda_{k,0}^{(1,1)} = \mu_k^2, \quad \lambda_{k,n}^{(1,2)} = \mu_k^2 - (c_n^1)^2, \quad \lambda_{k,n}^{(1,3)} = \mu_k^2 + (c_n^{(2)})^2,$$
$$\lambda_{k,0}^{(2,1)} = -\mu_k^2, \quad \lambda_{k,n}^{(2,2)} = -\mu_k^2 + (c_n^2)^2, \quad \lambda_{k,n}^{(2,3)} = -\mu_k^2 - (c_n^{(1)})^2,$$

bu yerda μ_k (12) tenglamaning (13) formula yordamida aniqlangan ildizlari, $c_n^{(1)}$ va $c_n^{(2)}$ lar esa $th(\alpha c) = -th(\beta c)$ -tenglamaning ildizlari bo‘lib, ular quyidagiga teng:

$$c_n^{(1)} = -\frac{\pi}{4\alpha} + \frac{\pi}{\alpha} n + O(e^{-2\pi n/\alpha}), \qquad c_n^{(2)} = -\frac{\pi}{4\beta} + \frac{\pi}{\beta} n + O(e^{-2\pi n/\beta}), \qquad n = 1, 2, \ldots.$$

Isbot. Qo‘yilgan masalaning xos sonlari (18) va (19) tenglamalarning ildizlaridir. (18) tenglama quyidagi

$$tg\left(a\sqrt{\mu_k^2 - \lambda}\right) = -th\left(\beta\sqrt{\mu_k^2 - \lambda}\right).$$

tenglamaga ekvivalentdir. $\mu_k^2 - \lambda$ ni c deb belgilaymiz, u holda 1-lemmaga muvofiq $th(\alpha c) = -th(\beta c)$ tengalama quyidagi ildizlarga ega:

$$c_0 = 0, \qquad c_n^{(1)} = -\frac{\pi}{4\alpha} + \frac{\pi}{\alpha} n + O(e^{-2\pi n/\alpha}),$$
$$ic_n^{(2)} = i(-\frac{\pi}{4\beta} + \frac{\pi}{\beta} n + O(e^{-2\pi n\alpha/\beta})), \; c_n^{(3)} = -(-\frac{\pi}{4\alpha} + \frac{\pi}{\alpha} n + O(e^{-2\pi n\beta/\alpha})),$$
$$ic_n^{(4)} = -i(-\frac{\pi}{4\beta} + \frac{\pi}{\beta} n + O(e^{-2\pi n\alpha/\beta})),$$

bu yerda $n \in N$.

U holda xos o‘z qiymatlar haqiqiy sonlar bo‘ladi, ya’ni

$$\lambda_{k,0}^{(1,1)} = \mu_k^2, \quad \lambda_{k,n}^{(1,2)} = \mu_k^2 - (c_n^{(1)})^2, \quad \lambda_{k,n}^{(1,3)} = \mu_k^2 + (c_n^{(2)})^2.$$
$$\lambda_{k,0}^{(2,1)} = -\mu_k^2, \quad \lambda_{k,n}^{(2,2)} = -\mu_k^2 + (c_n^{(2)})^2, \quad \lambda_{k,n}^{(2,3)} = -\mu_k^2 - (c_n^{(1)})^2.$$

Bularga mos keladigan xos funksiyalar quyidagi ko‘rinishga ega:

$$u_{k,n}^{(j,l)}(x,y) = X_k^{(j)}(x) Y_k^{(j)}(y, \lambda_{k,n}^{(j,l)}),$$

bu yerda

$$Y_k^{(j)}(y, \lambda_{k,n}^{(j,l)}) = Y_k^{(j)}(y)|_{\lambda = \lambda_{k,n}^{(j,l)}}, \qquad j = 1, 2, \qquad l = 1, 2, 3.$$

$\left\{X_k^{(1)}(x), X_k^{(2)}(x)\right\}$ sistema $L_2[-1,1]$ fazoda ortogonal emas. $\{(6),(8),(9)\}$ masalaga qo‘shma masala quyidagi ko‘rinishga ega:

$$\operatorname{sgn} x \cdot Z'' + d \cdot Z = 0, \; x \in (-1,0) \cup (0,1), \tag{20}$$
$$Z(0-0) = -Z(0+0), \quad Z'(0-0) = -Z'(0+0), \quad Z(-1) = Z(1) = 0. \tag{21}$$

$\{(20),(21)\}$-masalaning yechimlari quyidagi funksiyalardir:

$$Z_k^{(1)}(x) = \begin{cases} \dfrac{\sin[\mu_k(x-1)]}{\cos\mu_k}, & x > 0, \\ \dfrac{sh[\mu_k(x+1)]}{ch\mu_k}, & x < 0, \end{cases} \qquad Z_k^{(2)}(x) = \begin{cases} \dfrac{\sin[\mu_k(x-1)]}{\cos\mu_k}, & x > 0, \\ \dfrac{sh[\mu_k(x+1)]}{ch\mu_k}, & x < 0. \end{cases}$$

$\left\{Z_k^{(1)}; Z_k^2\right\}$ sistema $\left\{X_k^{(1)}, X_k^{(2)}\right\}$ sistemaga bioortogonal sistema hisoblanadi, ya’ni

$$\int_{-1}^{1} X_k^{(j)}(x)Z_m^{(l)}(x)dx = 0, \quad k \neq m, \quad j = 1,2; \; l = 1,2.$$

Quyidagi lemma o'rinlidir:

3-lemma. $L_2[-1,1]$ fazoda $\{Z_k^{(1)}; Z_k^2\}$ sistema to'ladir.

D **masala yechimining yagonaligi.** Aytaylik $\lambda \in R$, $\lambda \neq \lambda_{k,n}^{i,j}$ bo'lsin. Quyidagi funksiyalarni qaraymiz:

$$u_k^{(1)}(y) = \int_{-1}^{1} u(x,y)Z_k^{(1)}(x)dx, \qquad u_k^{(2)}(y) = \int_{-1}^{1} u(x,y)Z_k^{(2)}(x)dx, \qquad k = 1,2,3,.... \tag{22}$$

Ikkinchi tartibli $\frac{d^2}{dy^2}(u_k^{(1)}(y))$ hosilani hisoblaymiz va (1) tenglamaga asosan uni yozib olamiz, so'ngra ikki marta bo'laklab integrallab hamda chegaraviy shartlardan foydalangan keyin quyidagi tenglikka ega bo'lamiz:

$$signy \cdot \; (u_k^{(1)}"(y)) + (\lambda - d)u_k^{(1)}(y) = 0,$$

ya'ni (7) tenglamaga keladi. Shunday qilib, $u_k^{(1)}(y) \equiv Y_k^{(1)}(y)$, shuning uchun $u_k^{(1)}(y)$ (14) formula yordamida aniqlanadi. Xuddi shu narsa $u_k^{(2)}(y)$ funksiyalar uchun ham amal qiladi, ya'ni ular (15) formula yordamida aniqlanadi.

(5) chegaraviy shartlardan va (22) tenglikdan quyidagilarni topamiz:

$$\begin{cases} u_k^{(1)}(\beta) = \int_{-1}^{1} u(x,\beta)Z_k^{(1)}(x)dx = \int_{-1}^{1} \varphi(x)Z_k^{(1)}(x)dx = \varphi_k^{(1)}, \\ u_k^{(2)}(\beta) = \int_{-1}^{1} u(x,\beta)Z_k^{(2)}(x)dx = \int_{-1}^{1} \varphi(x)Z_k^{(2)}(x)dx = \varphi_k^{(2)}, \end{cases} \tag{23}$$

$$\begin{cases} u_k^{(1)}(-\alpha) = \int_{-1}^{1} u(x,-\alpha)Z_k^{(1)}(x)dx = \int_{-1}^{1} \psi(x)Z_k^{(1)}(x)dx = \psi_k^{(1)}, \\ u_k^{(2)}(-\alpha) = \int_{-1}^{1} u(x,-\alpha)Z_k^{(2)}(x)dx = \int_{-1}^{1} \psi(x)Z_k^{(2)}(x)dx = \psi_k^{(2)}. \end{cases} \tag{24}$$

U holda (14), (15), (23) va (24) ga asoslanib, $a_k^{(j)}, b_k^{(j)}$ larni topish uchun quyidagi algebraik tenglamalar sistemalariga kelamiz :

$$\begin{cases} a_k^{(1)}\, ch\left(\beta\sqrt{\mu_k^2 - \lambda}\right) + b_k^{(1)}\, sh\left(\beta\sqrt{\mu_k^2 - \lambda}\right) = \varphi_k^{(1)}, \\ a_k^{(1)}\cos\left(\alpha\sqrt{\mu_k^2 - \lambda}\right) - b_k^{(1)}\sin\left(\alpha\sqrt{\mu_k^2 - \lambda}\right) = \psi_k^{(1)}, \end{cases} \tag{25}$$

$$\begin{cases} a_k^{(2)}\cos\left(\beta\sqrt{\mu_k^2 + \lambda}\right) + b_k^{(2)}\sin\left(\beta\sqrt{\mu_k^2 + \lambda}\right) = \varphi_k^{(2)}, \\ a_k^{(2)}\, ch\left(\alpha\sqrt{\mu_k^2 + \lambda}\right) - b_k^{(2)}\, sh\left(\alpha\sqrt{\mu_k^2 + \lambda}\right) = \psi_k^{(2)}. \end{cases} \tag{26}$$

Agar barcha $k \in N$ lar uchun (18) va (19) munosabatlar bilan aniqlangan (25) va (26) sistemalarning determinantlari nolga teng bo'lmasa, bu sistemalar bir qiymatli yechiladi:

$$a_k^{(1)} = \frac{1}{\Delta_k^{(1)}(\alpha,\beta)}\left[\varphi_k^{(1)}\sin(\alpha\sqrt{\mu_k^2 - \lambda}) + \psi_k^{(1)} sh(\beta\sqrt{\mu_k^2 - \lambda}\right],$$

$$b_k^{(1)} = \frac{1}{\Delta_k^{(1)}(\alpha,\beta)}\left[\varphi_k^{(1)}\cos(\alpha\sqrt{\mu_k^2-\lambda}) - \psi_k^{(1)} ch(\beta\sqrt{\mu_k^2-\lambda}\right],$$

$$a_k^{(2)} = \frac{1}{\Delta_k^{(2)}(\alpha,\beta)}\left[\varphi_k^{(2)} sh(\alpha\sqrt{\mu_k^2+\lambda}) + \psi_k^{(2)}\sin(\beta\sqrt{\mu_k^2+\lambda}\right],$$

$$b_k^{(2)} = \frac{1}{\Delta_k^{(2)}(\alpha,\beta)}\left[\varphi_k^{(2)} ch(\alpha\sqrt{\mu_k^2+\lambda}) - \psi_k^{(2)}\cos(\beta\sqrt{\mu_k^2+\lambda}\right].$$

U holda topilgan $a_k^{(j)}, b_k^{(j)}$ qiymatlarni hisobga olgan holda $u_k^{(j)}(y)$ funksiya quyidagi ko‘rinishni oladi:

$$u_k^{(1)}(y) = \begin{cases} \frac{1}{\Delta_k^{(1)}(\alpha,\beta)}\left[\varphi_k^{(1)}\cdot\ \Delta_k^{(1)}(\alpha,y) + \psi_k^{(1)}\cdot\ sh((\beta-y)\sqrt{\mu_k^2-\lambda})\right], & y>0, \\ \frac{1}{\Delta_k^{(1)}(\alpha,\beta)}\left[\varphi_k^{(1)}\cdot\ \sin((\alpha+y)\sqrt{\mu_k^2-\lambda}) + \psi_k^{(1)}\cdot\ \Delta_k^{(1)}(-y,\beta)\right], & y<0, \end{cases} \quad (27)$$

$$u_k^{(2)}(y) = \begin{cases} \frac{1}{\Delta_k^{(2)}(\alpha,\beta)}\left[\varphi_k^{(2)}\cdot\ \Delta_k^{(2)}(\alpha,y) + \psi_k^{(2)}\sin((\beta-y)\sqrt{\mu_k^2+\lambda})\right], & y>0, \\ \frac{1}{\Delta_k^{(2)}(\alpha,\beta)}\left[\varphi_k^{(2)}\cdot\ sh((\alpha+y)\sqrt{\mu_k^2+\lambda}) + \psi_k^{(2)}\cdot\ \Delta_k^{(2)}(-y,\beta)\right], & y<0, \end{cases}$$

(28)

bu yerda

$$\Delta_k^{(1)}(\alpha,y) = \cos(\alpha\sqrt{\mu_k^2-\lambda})sh(y\sqrt{\mu_k^2-\lambda}) + \sin(\alpha\sqrt{\mu_k^2-\lambda})ch(y\sqrt{\mu_k^2-\lambda}),$$

$$\Delta_k^{(1)}(-y,\beta) = \cos(y\sqrt{\mu_k^2-\lambda})sh(\beta\sqrt{\mu_k^2-\lambda}) - \sin(y\sqrt{\mu_k^2-\lambda})ch(\beta\sqrt{\mu_k^2-\lambda}),$$

$$\Delta_k^{(2)}(\alpha,y) = ch(\alpha\sqrt{\mu_k^2+\lambda})\sin(y\sqrt{\mu_k^2+\lambda}) + sh(\alpha\sqrt{\mu_k^2+\lambda})\cos(y\sqrt{\mu_k^2+\lambda}),$$

$$\Delta_k^{(2)}(-y,\beta) = ch(y\sqrt{\mu_k^2+\lambda})\sin(\beta\sqrt{\mu_k^2+\lambda} - sh(y\sqrt{\mu_k^2+\lambda})\cos(\beta\sqrt{\mu_k^2+\lambda}).$$

Aytaylik $[-1,1]$ oraliqda $\varphi(x) = \psi(x) = 0$ bo‘lsin. (23), (24), (27) va (28) ga asoslanib quyidagiga ega bo‘lamiz:

$$\int_{-1}^{1} u(x,y)Z_k^{(j)}(x)dx = 0, \qquad j=1,2 \qquad k=1,2,3,....$$

$\left\{Z_k^{(1)}(x); Z_k^{(2)}(x)\right\}$ funksiyalar sistemasining $L_2[-1,1]$ da to‘laligiga asosan, $y\in[-\alpha,\beta]$ bo‘lgan har qanday y uchun $u(x,y)=0$ bo‘ladi. $u(x,y)$ funksiya $\bar{D}$ da uzluksiz bo‘lganligi sababli, $u(x,y)$ funksiya $\bar{D}$ da hamma joyda nolga teng, ya’ni $u(x,y)\equiv 0$.

Ayrim α,β va $k=p\in N$ uchun $\Delta_p^{(1)}(\alpha,\beta)=0$ yoki $\Delta_p^{(2)}(\alpha,\beta)=0$ bo‘lsin. Masalan, $\Delta_p^{(1)}(\alpha,\beta)=0$, lekin $\Delta_p^{(2)}(\alpha,\beta)\neq 0$ bo‘lsin. U holda (2)-(5) bir jinsli masala, ya’ni $\varphi(x)=\psi(x)=0$ bo‘lganda notrivial yechimga ega bo‘ladi:

$$u_p(x,y)=\begin{cases}\dfrac{\sin\left[\mu_p(x-1)\right]\Delta_p^{(1)}(\alpha,y)}{\cos\mu_p\cos(\alpha\sqrt{\mu_p^2-\lambda})}, & (x,y)\in D_1,\\ \dfrac{\sin\left[\mu_p(x-1)\right]\sin\left[(\alpha+y)\sqrt{\mu_p^2-\lambda}\right]}{\cos\mu_p\cos(\alpha\sqrt{\mu_p^2-\lambda})}, & (x,y)\in D_2,\\ \dfrac{sh\left[\mu_p(x+1)\right]\sin\left[(\alpha+y)\sqrt{\mu_p^2-\lambda}\right]}{ch\mu_p\cos(\alpha\sqrt{\mu_p^2-\lambda})}, & (x,y)\in D_3,\\ \dfrac{sh\left[\mu_p(x+1)\right]\Delta_p^{(1)}(\alpha,y)}{ch\mu_p\cos(\alpha\sqrt{\mu_p^2-\lambda})}, & (x,y)\in D_4.\end{cases}$$

Agar $k=p\in N$ bo‘lganda $\Delta_{k_0}^{(2)}(\alpha,\beta)=0$ bo‘lsa, u holda masalaning trivial bo‘lmagan yechimi ham mavjud bo‘ladi.

Tabbiy savol paydo bo‘lishi mumkin, ya’ni $\Delta_k^{(j)}(\alpha,\beta)$ determinant nolga teng bo‘lishi mumkinmi yoki yoqmi ? Ularni quyidagicha tasvirlab olamiz:

$$\left.\begin{aligned}\Delta_k^{(1)}(\alpha,\beta)=\Delta_k^{(1)}(\alpha,\beta)=\sqrt{ch(2\beta\sqrt{\mu_k^2-\lambda}}\sin(\alpha\sqrt{\mu_k^2-\lambda}+\xi_k),\\ \Delta_k^{(2)}(\alpha,\beta)=\Delta_k^{(1)}(\alpha,\beta)=\sqrt{ch(2\alpha\sqrt{\mu_k^2-\lambda}}\sin(\beta\sqrt{\mu_k^2-\lambda}+\chi_k),\end{aligned}\right\}\tag{29}$$

bu yerda $\xi_k=arctg(th(\beta\sqrt{\mu_k^2-\lambda})),\quad \chi_k=arctg(th(\alpha\sqrt{\mu_k^2-\lambda}))$ bo‘lib, ular uchun quyidagi munosabat o‘rinli:

$$\lim_{k\to\infty}\xi_k=\lim_{k\to\infty}\chi_k=\frac{\pi}{4}$$

va barcha $k\in N$ uchun $\xi_k<\frac{\pi}{4}$ va $\chi_k<\frac{\pi}{4}$ tengsizliklar bajariladi. Shundan ularning nol to‘plamlarini topamiz: Bu yerdan ularning nollari to‘plamini topamiz:

$$\alpha_{k,m}=\frac{\pi m-\xi_k}{\sqrt{\mu_k^2-\lambda}},\qquad \beta_{k,t}=\frac{\pi t-\chi_k}{\sqrt{\mu_k^2-\lambda}},\qquad m,t,k\in N.\tag{30}$$

(30) tenglik $\alpha_{k,m}$ va $\beta_{k,t}$ ga nisbatan sistema tashkil qiladi. Ushbu sistemaning muvofiqligiga ishonch hosil qilish mumkin, bundan tashqari, uning (α,β) yechimlari son-sanoqsizdir. Shu sababli, quyidagi teorema o‘rinlidir.

Teorema 1: Agar (2)-(5) masalaning yechimi mavjud bo‘lsa, u holda u yagona bo‘ladi, faqat agar barcha $k\in N$ uchun $\Delta_k^{(j)}(\alpha,\beta)\neq 0,\ j=1,2$ shartlar bajarilsa.

Yechimning mavjudligi. (27) va (28) formulalardan ko‘rinib turibdiki $\Delta_k^{(j)}$ ifodalar qiymatlarining maxrajlari hisoblanadi va (30) ni qanoatlantiradigan α,β qiymatlarida nolga aylanishi mumkin, ya’ni “kichik maxrajlar” muammosi paydo bo‘ladi. Shuning uchun (2)-(5) masalaning yechim mavjudligini asoslash asoslash uchun katta k ning qiymatlarida $\Delta_k^{(j)}$ ifodalari nolga aylanmaydigan α va β sonlarining mavjudligini ko‘rsatish kerak.

Lemma 4. Agar quyidagi shartlardan biri bajarilsa:

1. α har qanday natural son bo'lib, $4p-3$ ko'rinishidagi sonlar bundan mustasno, bu yerda $p \in N$,

2. α har qanday kasr son bo'lib, ya'ni $\alpha = \frac{p}{q}$, bu yerda $p,q \in N$ $(p,q)=1$ va $q-p$ soni 4 ga karrali emas, u holda shunday $C_{01} > 0$ doimiy va $k_{01} \in N$ sonlar mavjud bo'lib, barcha $k > k_{01}$ uchun quyidagi

$$\left|\Delta_k^{(1)}(\alpha,\beta)\right| \geq C_{01} e^{\pi k \beta} \tag{31}$$

baho o'rinli bo'ladi.

Isbot. (29) formuladan quyidagini topamiz:

$$\sqrt{ch(2\beta\sqrt{\mu_k^2-\lambda}} \geq Ce^{\beta\sqrt{\mu_k^2-\lambda}} \geq \overline{C} e^{\pi k \beta},$$

bu yerda $\overline{C}$ - musbat o'zgarmas kattalik.

Shunday qilib,

$$\left|\sin(\alpha\sqrt{\mu_k^2-\lambda}+\xi_k)-\sin(\alpha(-\frac{\pi}{4}+\pi k)+\frac{\pi}{4})\right| \leq \left|(\alpha\sqrt{\mu_k^2-\lambda}+\frac{\pi}{4}-\pi k)+\xi_k-\frac{\pi}{4})\right| \leq$$

$$\leq \left|\alpha(\pi k-\frac{\pi}{4}-\sqrt{\mu_k^2-\lambda})\right| = \alpha\left|\frac{(\pi k-\frac{\pi}{4})^2-\mu_k^2+\lambda}{\pi k-\frac{\pi}{4}+\sqrt{\mu_k^2-\lambda}}\right| \leq M_{1\alpha} k e^{-2\pi k}, \qquad M_{1\alpha} > 0,$$

Bu ifodani baholaymiz

$$\left|\sin\left[\alpha(-\frac{\pi}{4}+\pi k)+\frac{\pi}{4}\right]\right| = |z_k|.$$

$\alpha \in N$ bo'lsin. Uch holat mavjud:

1) Aytaylik $\lambda \in N$ bo'lsin. $\alpha = 2p,\ \alpha = 4p-1,\ \alpha = 4p-3,$ bu yerda $p \in N$. Dastlabki ikki holda $|z_k| = const > 0$ bo'ladi, uchinchi holda esa $|z_k| = 0.$

2) Aytaylik $\alpha = \frac{p}{q}$ bo'lsin, bunda $p,q \in N,\ (p,q)=1$. U holda

$$z_k = \sin\left[\frac{\pi}{4}(1-\frac{p}{q})+\pi\frac{kp}{q}\right].$$

kp sonini qoldiq q ga qoldiq bilan bo'lamiz: $kp = sq + r$, bu yerda $s,r \in N \cup \{0\}$, $0 \leq r < q$. U holda $z_k = \sin \pi(q-p+4r)/4q$ va $q-p$ 4 ga karrali bo'lmagan sharoitda, $|z_k| = const > 0$ bo'ladi. □

Agar $\alpha = 4p-3,\ p \in N, p \geq 2$ bo'lsa, unda mavjudligini isbotlash oson bo'lgan $C'_{01} > 0$ va $k'_{01} \in N$ konstantalar mavjud , shunday qilib , barcha $k > k'_{01}$ uchun quyidagi baholar to'g'ri bo'ladi

$$\left|\Delta_k^{(2)}(\alpha,\beta)\right| \geq C'_{01} e^{-\pi k \beta}, \qquad 0 < \beta \leq 1,$$

$$\left|\Delta_k^{(2)}(\alpha,\beta)\right| \geq C'_{01} e^{\pi k(\beta-2)}, \qquad \beta > 1.$$

Shunday qilib, **lemma 4** da keltirilgan $\alpha = \frac{4}{p} - 3$ sharti zaruriy hisoblanadi.

Lemma 5. Agar quyidagi shartlardan biri bajarilsa:

1. β - bu $4p-3$ ko'rinishdagi sonlardan tashqari, istalgan natural son, $p \in N$,

2. β - bu har qanday ratsional son, ya'ni $\beta = \frac{p}{q}$, bu yerda $p, q \in N,\ (p,q)=1,\ q-p$ soni 4 ga karrali emas, u holda $C_{02} > 0$ doimiy va $k_{02} \in N$ bo'lgan shunday k soni mavjudki, barcha $k > k_{02}$ uchun quyidagi baho o'rinli:

$$\left|\Delta_k^{(2)}(\alpha,\beta)\right| \geq Ce^{\pi k\alpha}. \tag{32}$$

Agar (31) va (32) baholar hamda $\Delta_k^{(j)}(\alpha,\beta) \neq 0$ sharti $k \neq k_0$ da bajarilgan bo'lsa, u holda (2)-(5)-masalaning yechimini

$$u(x,y) = \sum_{k=1}^{\infty} u_k^{(1)}(y)X_k^{(1)}(x) + u_k^{(2)}(y)X_k^{(2)}(x) \tag{33}$$

Furye qatori ko'rinishida tasvirlash mumkin.

$\varphi(x)$ va $\psi(x)$ uchun shunday shartlarni aniqlaymizki, (33) qator D sohada tekis yaqinlashib, x va y bo'yicha hadma-had differensiallashga imkon beradi.

Quyidagi munosabatlarni ko'rib chiqamiz.

$$P_k^{(j)} = \frac{\Delta_k^{(j)}(\alpha,y)}{\Delta_k^{(j)}(\alpha,\beta)}, \qquad Q_k^{(1)}(y) = \frac{sh((\beta-y)\sqrt{\mu_k^2-\lambda}}{\Delta_k^{(1)}(\alpha,\beta)},$$

$$Q_k^{(2)}(y) = \frac{\sin((\beta-y)\sqrt{\mu_k^2+\alpha}}{\Delta_k^{(2)}(\alpha,\beta)},\ M_k^{(1)}(y) = \frac{\sin((\alpha+y)\sqrt{\mu_k^2-\lambda}}{\Delta_k^{(1)}(\alpha,\beta)},$$

$$M_k^{(2)}(y) = \frac{sh((\alpha+y)\sqrt{\mu_k^2+\lambda}}{\Delta_k^{(2)}(\alpha,\beta)},\ N_k^{(j)}(y) = \frac{\Delta_k^{(j)}(-y,\beta)}{\Delta_k^{(j)}(\alpha,\beta)},$$

bu yerda birinchi uch ifoda $y > 0$ bo'lgan holatda aniqlangan, oxirgi uchta ifoda esa $y < 0$ bo'lgan holatda aniqlangan.

Lemma 6. Agar (31) va (32) baholar barcha $k > k_0$ uchun bajarilgan bo'lsa, unda bunday k uchun quyidagi baholar to'g'ri bo'ladi:

$$\left|P_k^{(j)}(y)\right| \leq C_1^{(j)}, \qquad \left|(P_k^{(j)}(y))'\right| \leq C_2^{(j)}k, \quad \left|(P_k^{(j)}(y))''\right| \leq C_3^{(j)}k^2,$$

$$\left|Q_k^{(1)}(y)\right| \leq C_4^{(1)}, \qquad \left|(Q_k^{(1)}(y))'\right| \leq C_5^{(1)}k, \quad \left|(Q_k^{(1)}(y))''\right| \leq C_6^{(1)}k^2,$$

$$\left|Q_k^{(2)}(y)\right| \leq C_4^{(2)}e^{-\pi k\alpha}, \qquad \left|(Q_k^{(2)}(y))'\right| \leq C_5^{(2)}ke^{-\pi k\alpha}, \quad \left|(Q_k^{(2)}(y))''\right| \leq C_6^{(2)}k^2e^{-\pi k\alpha},$$

$$\left|M_k^{(1)}(y)\right| \leq C_7^{(1)}e^{-\pi k\beta}, \qquad \left|(M_k^{(1)}(y))'\right| \leq C_8^{(1)}ke^{-\pi k\beta}, \quad \left|(M_k^{(1)}(y))''\right| \leq C_9^{(1)}k^2e^{-\pi k\beta},$$

$$\left|M_k^{(2)}(y)\right| \leq C_7^{(2)}, \qquad \left|(M_k^{(2)}(y))'\right| \leq C_8^{(2)}k, \quad \left|(M_k^{(2)}(y))''\right| \leq C_9^{(2)}k^2,$$

$$\left|N_k^{(j)}(y)\right| \leq C_{10}^{(j)}, \qquad \left|(N_k^{(j)}(y))'\right| \leq C_{11}^{(j)}k, \quad \left|(N_k^{(j)}(y))''\right| \leq C_{12}^{(j)}k^2,$$

bu yerda $C_l^j > 0,\ j=1,2,\ l=\overline{1,12}$.

Lemma 7. Har qanday $k > k_0$ uchun quyidagi baholar to'g'ri bo'ladi:

$$\left|u_k^{(j)}(y)\right| \leq C_{13}^{(j)}(\left|\varphi_k^{(j)}\right| + \left|\psi_k^{(j)}\right|),\ \left|(u_k^{(j)}(y))'\right| \leq C_{14}^{(j)}k(\left|\varphi_k^{(j)}\right| + \left|\psi_k^{(j)}\right|),$$

$$\left|(u_k^{(j)}(y)'')\right| \leq C_{15}^{(j)}k^2(\left|\varphi_k^{(j)}\right| + \left|\psi_k^{(j)}\right|),\ \ y \in [-\alpha,\beta],$$

bu yerda $C_l^{(j)} > 0,\ j=1,2,\ l=\overline{13,15}.$

7- lemmaga ko'ra (33)-qator va uning hosilalari D sohada, ikkinchi tartibli hosilalari esa D_i sohada mavjud, $i=1,4$ da quyidagi sonli qator bilan majorantlanadi:

$$C\sum_{k=1}^{\infty}k^2(\left|\varphi_k^{(1)}\right|+\left|\psi_k^{(1)}\right|+\left|\varphi_k^{(2)}\right|+\left|\psi_k^{(2)}\right|, \quad C=const>0.$$

8-lemma. Agar $\varphi(x)$ va $\psi(x)$ funksiyalar $C^1[-1,1]\cap C^3[-1,0]\cap C^3[0,1]$ sinfga tegishli bo'lib, va ushbu oraliqda to'rtinchi tartibli hosilaga ega bo'lsa, ya'ni

$$\varphi(-1)=\varphi(1)=\varphi''(-1)=\varphi''(1)=0,$$
$$\psi(-1)=\psi(1)=\psi''(-1)=\psi''(1)=0,$$
$$\varphi''(0+0)=-\varphi''(0-0),$$
$$\varphi'''(0+0)=-\varphi'''(0-0),$$
$$\psi''(0+0)=-\psi''(0-0),$$
$$\psi'''(0+0)=-\psi'''(0-0),$$

bo'lganda quyidagi munosabatlar o'rinlidir:

$$\varphi_k^{(1)}=\frac{p_k^{(1)}}{\mu_k^4}, \quad \varphi_k^{(2)}=\frac{p_k^{(2)}}{\mu_k^4}, \quad \psi_k^{(1)}=\frac{q_k^{(1)}}{\mu_k^4}, \quad \varphi_k^{(1)}=\frac{q_k^{(2)}}{\mu_k^4},$$

bu yerda

$$p_k^{(1)}=\int_{-1}^{1}\varphi^{(4)}(x)Z_k^{(1)}(x)dx, \qquad p_k^{(2)}=\int_{-1}^{1}\varphi^{(4)}(x)Z_k^{(2)}(x)dx,$$
$$q_k^{(1)}=\int_{-1}^{1}\psi^{(4)}(x)Z_k^{(1)}(x)dx, \qquad q_k^{(2)}=\int_{-1}^{1}\psi^{(4)}(x)Z_k^{(2)}(x)dx.$$

Isbotlash uchun (23) va (24) tengliklarni integrallarni to'rt marta bo'laklab integrallaymiz.

$Z_k^{(1)}, Z_k^{(2)}$ sistema uchun Bessel tenglamasi to'g'ri ekanligini ko'rsatish mumkin chunki $\varphi^{(4)}(x)$ va $\psi^{(4)}(x)$ bo'lakli uzluksiz funksiyalar bo'lsa, $\sum_{k=1}^{\infty}(p_k^{(j)})^2$ va $\sum_{k=1}^{\infty}(q_k^{(j)})^2$ qatorlar yaqinlashadi. Bundan tashqari quyidagi qator ham yaqinlashuvchi

$$C\sum_{k=1}^{\infty}\frac{1}{k}(\left|p_k^{(1)}\right|+\left|q_k^{(1)}\right|+\left|p_k^{(2)}\right|+\left|q_k^{(2)}\right|),$$

bo'lganligi uchun (33) qator va x va y o'zgaruvchilarga nisbatan differensiallangan qatorlarning tekis yaqinlashuvchi ekanligi, shuningdek, D_i sohada ikkinchi tartibli hosilalarning yig'indisidan hosil bo'lgan qatorlarning ham tekis yaqinlashuvchi ekanligi kelib chiqadi. Agar 4-5 lemmalarda ko'rsatilgan α va β sonlari va $k=k_1,k_2...,k_l$, $1\le k_1<k_2<...<k_l\le k_0$ bo'lsa, va $\Delta^{(j)}k(\alpha,\beta)=0$ (masalan, aniqlik uchun $\Delta^{(1)}k(\alpha,\beta)=0$, $\Delta^{(2)}k(\alpha,\beta)\neq 0$ shartlarga to'g'ri kelsa, unda (16) sistemaning $a_k^{(1)}$ va $b_k^{(1)}$ bo'yicha yechimlarining mavjudligi uchun zarur va yetarli shartlar bajarilishi kerak, ya'ni

$$\varphi_{k_i}^{(1)}\cos(\alpha\sqrt{\mu_k^2-\lambda}=\psi_{k_1}^{(1)}ch(\beta\sqrt{\mu_k^2-\lambda}), \qquad i=\overline{1,l}. \tag{34}$$

u holda $k=k_1,k_2...,k_l$ lar uchun quyidagiga ega bo'lamiz:

$$\tilde{u}_k(x,y)=\begin{cases}\dfrac{\sin\left[\mu_k(x-1)\right]}{\cos\mu_k}(ch(y\sqrt{\mu_k^2-\lambda}+\\ \qquad +\dfrac{\varphi_k^{(1)}-ch(\beta\sqrt{\mu_k^2-\lambda})}{sh(\beta\sqrt{\mu_k^2-\lambda})}sh(y\sqrt{\mu_k^2-\lambda}) & (x,y)\in D_1,\\ \dfrac{\sin\left[\mu_k(x-1)\right]}{\cos\mu_k}(\cos(y\sqrt{\mu_p^2-\lambda})+\\ \qquad +\dfrac{\varphi_k^{(1)}-ch(\beta\sqrt{\mu_k^2-\lambda})}{sh(\beta\sqrt{\mu_k^2-\lambda})}\sin\left[y\sqrt{\mu_k^2-\lambda}\right], & (x,y)\in D_2,\\ \dfrac{sh\left[\mu_k(x-1)\right]}{ch\mu_k}(\cos(y\sqrt{\mu_k^2-\lambda})+\\ \qquad +\dfrac{\varphi_k^{(1)}-ch(\beta\sqrt{\mu_k^2-\lambda})}{sh(\beta\sqrt{\mu_k^2-\lambda})}\sin\left[y\sqrt{\mu_k^2-\lambda}\right], & (x,y)\in D_3,\\ \dfrac{sh\left[\mu_k(x-1)\right]}{ch\mu_k}(ch(y\sqrt{\mu_k^2-\lambda})+\\ \qquad +\dfrac{\varphi_k^{(1)}-ch(\beta\sqrt{\mu_k^2-\lambda})}{sh(\beta\sqrt{\mu_k^2-\lambda})}sh(y\sqrt{\mu_k^2-\lambda})), & (x,y)\in D_4.\end{cases}$$

Demak, (2), (5) masalaning yechimi quyidagi qatorlar yig'indisi sifatida aniqlanadi:

$$u(x,y)=(\sum_{k=1}^{k_1-1}+\sum_{k=1}^{k_1-1}+....\sum_{k=k_1+1}^{\infty})u_k^{(1)}(y)X_k^{(1)}(x)+$$

$$+\sum_{k=k_1,k_2...,k_l}A_k\tilde{u}(x,y)+\sum_{k=1}^{\infty}u_k^{(2)}(y)X_k^{(1)}(x),$$

(35)

bu yerda A_k— ixtiyoriy koeffitsiyentlar bo'lib, agar yig'indining quyi chegarasi yuqori chegaradan katta bo'lsa, yakuniy yig'indilarni nol deb hisoblash kerak. Agar ba'zi k qiymatlarida $\Delta^{(2)}k(\alpha,\beta)=0,\quad \Delta^{(1)}k(\alpha,\beta)\neq 0$ yoki agar ikkala maxraj ham nolga teng bo'lsa, o'xshash yechimlar quriladi. Shunday qilib, quyidagi teorema isbotlangan. **Teorema 2**. Agar $\varphi(x)$,$\psi(x)$ funksiyalar 8-lemmaning shartlarini qanoatlantirsa va $k>k_0$ bo‘lganda (31), (32) baholar o‘rinli bo‘lsa 4 va 5-lemma ko‘rsatilgan α va β qiymatlarida barcha $k=\overline{1,k_0}$ uchun $\Delta^{(1)}k(\alpha,\beta)\neq 0$,$\Delta^{(2)}k(\alpha,\beta)\neq 0$ shartlari bajarilsa, u holda (2)–(5) Dirixle masalasining yagona yechimi mavjud va u (33) qator orqali aniqlanadi aniqlanadi. Shu bilan Dirixle masalasining tadqiqoti yakunlandi.

XULOSA

Olingan natijalarga ko‘ra D tenglamaning yechimi mavjud va yagona ekan. Bu yechimni mavjudligini isbotlash uchun katta k ning qiymatlarida $\Delta_k^{(j)}$ ifodalari nolga aylanmaydigan α va β sonlarining mavjudligini ko‘rsatildi, yechimning yagonaligini ko‘rsatishda esa (25) va (26) sistemalarning determinantlari nolga teng bo‘lmaganda bu sistemalar bir qiymatli yechilishidan foydalanildi.

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