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VAQT YO'NALISHLARI TURLICHA BO'LGAN PARABOLO-GIPERBOLIK TENGLAMA
UCHUN CHEGARAVIY MASALA

КРАЕВАЯ ЗАДАЧА ДЛЯ УРАВНЕНИЙ ПАРАБОЛО-ГИПЕРБОЛИЧЕСКОГО ТИПА С
РАЗНЫМИ НАПРАВЛЕНИЯМИ ВРЕМЕНИ

BOUNDARY PROBLEM FOR A PARABOLIC-HYPERBOLIC EQUATIONS WITH
DIFFERENT TIME DIRECTIONS

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Annotatsiya

Mazkur ishda vaqt yo'nalishlari turlicha bo'lgan aralash tipdagi bir tenglama uchun qo'yilgan chegaraviy masala yechimining mavjudligi va yagonalini isbotlangan.

Аннотация

В настоящей работе сформулирована краевая задача для уравнения парабола-гиперболического типа с различными направлениями времени и доказано существование и единственность задачи.

Abstract

In the present work, a boundary-value problem has been formulated for a parabolic-hyperbolic equation with different time directions and the existence and uniqueness of the problem has been proved.

Kalit so'zlar: Aralash tipdagi tenglama, chegaraviy masala, integral tenglamalar usuli.

Ключевые слова: Уравнение смешанного типа, краевая задача, метод интегральных уравнений.

Key words: Equation of mixed type, boundary value problem, method of integral equations.

KIRISH

xOy tekisligining $D = D_0 \cup D_1 \cup D_2$ sohasida ushbu

$$0 = Lu \equiv \begin{cases} u_{xx} - u_t = 0, & (x, t) \in D_0, \\ u_{xx} - u_{tt} = 0, & (x, t) \in D_1, \\ u_{tt} + u_x = 0, & (x, t) \in D_2 \end{cases} \quad (1)$$

tenglamani qaraylik, bu yerda $D_0 = \{(x, t) : 0 < x < +\infty, 0 < t < 1\}$,

$D_1 = \{(x, t) : -x < t < x - 1, 0 < x < 1/2\}$, $D_2 = \{(x, t) : -1 < x < 0, 0 < t < 1\}$.

Ushbu $I_1 = \{(x, t) : -1 < x < 0, t = 0\}$, $I_2 = \{(x, t) : 0 < t < 1, x = 0\}$

belgilashlarni kiritib, (1) tenglama uchun D sohada quyidagi masalani o'rganamiz:

1-masala. $Lu = 0$ tenglamaning

$$u(x, 0) = \varphi_1(x), \quad 1 \leq x < +\infty; \quad (2)$$

$$u(x, 0) = \varphi_2(x), \quad -1 \leq x \leq 0; \quad (3)$$

$$u(x, 1) = \varphi_3(x), \quad -1 \leq x \leq 0; \quad (4)$$

$$\lim_{x \rightarrow +\infty} u(x, t) = 0, \quad 0 \leq t \leq 1; \quad (5)$$

$$u(x, -x) = \psi_1(x), \quad 0 \leq x \leq 1/2; \quad (6)$$

$$u(x, x-1) = \psi_2(x), \quad 1/2 \leq x \leq 1 \quad (7)$$

shartlarni qanoatlantiruvchi $u(x, y) \in C(\bar{D}) \cap C^1(D_0 \cup D_1 \cup D_2 \cup I_1)$ regulyar yechimi topilsin, bu yerda $\varphi_i(x)$, $i = \overline{1,3}$ va $\psi_j(x)$, $j = \overline{1,2}$ berilgan funksiyalar.

Masalaning tadqiqoti. Qo'yilgan masala yechimining mavjudligi va yagonalini isbotlaymiz. Shu maqsadda, masala shartlariga asoslanib, quyidagi belgilashlarni kiritaylik:

$$u(x, -0) = u(x, +0) = \tau_1(x), \quad 0 \leq x \leq 1; \quad (8)$$

$$u(-0, t) = u(+0, t) = \tau_2(t), \quad 0 \leq t \leq 1; \quad (9)$$

$$\lim_{x \rightarrow +0} u_x(x, t) = \lim_{x \rightarrow -0} u_x(x, t) = \tau_3(t), \quad 0 < t < 1, \quad (10)$$

bu yerda $\tau_1(x)$, $\tau_2(t)$, $\tau_3(t)$ - noma'lum funksiyalar.

U holda masala yechimini D_1 sohada $u_{xx} - u_{tt} = 0$ tenglama uchun (6) va $u(x, +0) = \tau_1(x)$, $x \in [0; 1]$ shartlarni qanoatlantiruvchi yechimi sifatida ushbu

$$u(x, t) = \tau_1(x+t) + \psi_1\left(\frac{m+x-t}{2}\right) - \psi_1\left(\frac{m+x+t}{2}\right) \quad (11)$$

ko'rinishda aniqlashimiz mumkin [1]. (11) yechimni (7) shartga bo'ysundirib, noma'lum $\tau_1(x)$ funksiyani ushbu ko'rinishda topamiz:

$$\tau_1(x) = \psi_1\left(\frac{x+1}{2}\right) + \psi_2\left(\frac{x+1}{2}\right) - \psi_1(1), \quad 0 \leq x \leq 1. \quad (12)$$

Endi masalani D_0 sohada qaraylik. Ma'lumki, $u_{xx} - u_t = 0$ tenglamaning D_0 sohaning yopig'ida aniqlangan, uzluksiz hamda (2), (5) va ushbu

$$u(x, 0) = \tau_1(x), \quad 0 \leq x \leq 1; \quad (12)$$

$$\lim_{x \rightarrow +0} u_x(x, t) = \tau_3(t), \quad 0 < t < 1; \quad (13)$$

shartlarni qanoatlantiruvchi yechimi ushbu

$$u(x, t) = \int_0^1 \frac{\sqrt{x\xi}}{2t} I_{-1/2}\left(\frac{x\xi}{2t}\right) e^{-(x^2+\xi^2)/4t} \tau_1(\xi) d\xi + \int_1^{+\infty} \frac{\sqrt{x\xi}}{2t} I_{-1/2}\left(\frac{x\xi}{2t}\right) e^{-(x^2+\xi^2)/4t} \varphi_1(\xi) d\xi - \frac{1}{\sqrt{\pi}} \int_0^t \tau_3(\eta) (t-\eta)^{-1/2} e^{-x^2/4(t-\eta)} d\eta \quad (14)$$

ko'rinishda aniqlanadi [2], bu yerda $I_p(z)$ - mavhum argumentli Bessel funksiyasi bo'lib [3,4], quyidagi ko'rinishda aniqlanadi:

$$I_p(z) = \sum_{k=0}^{+\infty} \frac{(z/2)^{2k-1/2}}{k! \Gamma(k+1/2)}.$$

(14) tenglikda x ni nolga intiltirib va (9) belgilashni e'tiborga olib, noma'lum $\tau_2(t)$ va $\tau_3(t)$ funksiyalar orasidagi D_0 sohadan olingan quyidagi

$$\tau_2(t) = -\frac{1}{\sqrt{\pi}} \int_0^t \tau_3(\eta) (\eta-t)^{-1/2} d\eta + \Phi_1(t), \quad 0 \leq t \leq 1 \quad (15)$$

munosabatga ega bo'lamiz, bu yerda

$$\Phi_1(t) = (\pi t)^{-1/2} \left[\int_0^1 \tau_1(\xi) e^{-\xi^2/4t} d\xi + \int_1^{+\infty} \varphi_1(\xi) e^{-\xi^2/4t} d\xi \right].$$

Endi masala shartlarini e'tiborga olib, $u_{xx} + u_t = 0$ tenglama va (2), (3)

shartlarda x ni nolga intiltiramiz. Natijada $\tau_2(t)$ va $\tau_3(t)$ noma'lumlar orasida ushbu

$$\tau_2''(t) + \tau_3(t) = 0, \quad 0 < t < 1 \quad (16)$$

ko'rinishdagi ikkinchi munosabatga va

$$\tau_2(0) = \varphi_1(0), \quad \tau_2(1) = \varphi_3(0), \quad (17)$$

shartlarga ega bo'lamiz.

Agar (15), (16) va (17) munosabatlardan foydalanib, $\tau_2(t)$ va $\tau_3(t)$ noma'lum funksiyalarni bir qiymatli topsak, u holda D_0 sohada masalaning yechimi (14) formula orqali, D_2 sohada esa $u_{xx} + u_t = 0$ tenglama uchun birinchi chegaraviy masalaning yechimi sifatida [4]

$$u(x, t) = \int_0^1 \tau_2(\eta) G_1(x, t; 0, \eta) d\eta + \int_x^0 \varphi_2(\xi) G_{1\eta}(x, t; \xi, 0) d\xi - \int_x^0 \varphi_3(\xi) G_{1\eta}(x, t; \xi, 1) d\xi$$

(18)

ko'rinishda aniqlanadi [5,7], bu yerda

$$G_1(x, t; \xi, \eta) = \frac{1}{\sqrt{\pi(\xi - x)}} \sum_{n=-\infty}^{\infty} \left\{ \exp \left[-\frac{(t - \eta - 2n)^2}{4(\xi - x)} \right] - \exp \left[-\frac{(t + \eta - 2n)^2}{4(\xi - x)} \right] \right\}.$$

Shuning uchun bundan buyon { (15), (16), (17) } tenglamalar sistemasini yechish bilan shug'ullanamiz. Kasr tartibli integral operator $D_{0t}^{-\alpha}$ ning ko'rinishidan foydalanib, (15) tenglikni quyidagi

$$\tau_2(t) = -D_{0t}^{-1/2} \tau_3(t) + \Phi_1(t), \quad 0 \leq t \leq 1$$

ko'rinishida yozib olamiz. Bu tenglikka $D_{0t}^{1/2}$ differensial operatorni tatbiq qilib va $D_{0t}^{-1/2} D_{0t}^{1/2} \tau_3(t) = \tau_3(t)$ tenglikni hisobga olib [], ushbu

$$\tau_3(t) = -D_{0t}^{1/2} \tau_2(t) + D_{0t}^{1/2} \Phi_1(t), \quad 0 < t < 1 \quad (19)$$

tenglikka ega bo'lamiz. Buni (14) tenglamaga qo'yib, quyidagi

$$\tau_2''(t) - D_{0t}^{1/2} \tau_2(t) = -D_{0t}^{1/2} \Phi_1(t), \quad 0 < t < 1 \quad (20)$$

integro-differensial tenglamani hosil qilamiz.

(20) tenglamaning (17) shartlarni qanoatlantiruvchi yechimi mavjudligini va yagonaligini isbotlaylik. Avval bir jinsli masalani, ya'ni

$$\tau_2''(t) - D_{0t}^{1/2} \tau_2(t) = 0, \quad 0 < t < 1; \quad (21)$$

$$\tau_2(0) = \tau_2(1) = 0 \quad (22)$$

masalani qaraymiz. Faraz qilaylik, bu masala $\tau_2(t) \neq 0$, $0 \leq t \leq 1$ yechimga ega bo'lsin.

Unda $\sup |\tau_2(t)| = |\tau_2(\xi)| \neq 0$, $\xi = const \in [0; 1]$ bo'ladi. (22) shartlarga asosan $\xi \in (0, 1)$. U holda $\tau_2(t)$ funksiya $t = \xi$ nuqtada musbat maksimumga yoki manfiy minimumga erishadi. Buni hamda butun hosilalarning xossalarni va kasr tartibli differensial operator uchun ekstremum prinsipini e'tiborga olsak [8], $\tau_2(\xi)$ – musbat maksimum (manfiy minimum) bo'lganda

$$\tau_2''(\xi) \leq 0 (\geq 0), \quad D_{0t}^{1/2} \tau_2(t) \Big|_{t=\xi} > 0 (< 0)$$

tengsizliklar o'rinli bo'ladi. Bularga ko'ra

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$$\tau_2''(\xi) - D_{0t}^{1/2} \tau_2(t) \Big|_{t=\xi} > 0 (< 0)$$

tengsizlik ham o'rinli bo'ladi. Bu tengsizlik (21) tenglamaga ziddir. Hosil bo'lgan bu qarama-qarshilik $\tau_2(t) \neq 0, 0 \leq t \leq 1$ degan farazimiz noto'g'ri ekanligini ko'rsatadi. Demak, {(21), (22)} masala faqat trivial yechimga ega ekan. Bundan agar {(17),(20)} masalaning yechimi mavjud bo'lsa, uning yagona bo'lishi kelib chiqadi.

Endi {(17),(20)} masala yechimining mavjudligini ko'rsatishga o'tamiz. (20) tenglikni

$$\tau_2''(t) = D_{0t}^{1/2} \tau_2(t) - D_{0t}^{1/2} \Phi_1(t), \quad 0 < t < 1$$

ko'rinishida yozib, uning o'ng tomonini ma'lum funksiya deb hisoblasak, {(17),(20)} masalaning yechimini

$$\tau_2(t) = \int_0^1 H(t,\eta) D_{0\eta}^{1/2} [\tau_2(\eta) - \Phi_1(\eta)] d\eta + [\varphi_3(0) - \varphi_1(0)]t + \varphi_1(0), \quad t \in [0,1]$$

ko'rinishida yozish mumkin [6,8], bu yerda

$$H(t,\eta) = \begin{cases} t(\eta - 1), & 0 \leq t \leq \eta; \\ \eta(t - 1) & \eta \leq t \leq 1. \end{cases}$$

Bu tenglikda $\varphi_2(0) = 0$ ekanligini hisobga olib va $D_{0\eta}^{1/2} f(\eta)$ ifoda o'rniga uning

$$D_{0\eta}^{1/2} f(\eta) \equiv \frac{1}{\sqrt{\pi}} \frac{d}{d\eta} \int_0^\eta (\eta - z)^{-1/2} f(z) dz$$

yoyilmasini qo'yib, so'ngra hosil bo'lgan integralni bo'laklab,

$$\tau_2(t) = \tilde{\Phi}_1(t) - \int_0^1 \tau_2(z) K_1(t,z) dz, \quad 0 \leq t \leq 1 \quad (23)$$

ko'rinishdagi integral tenglamaga ega bo'lamiz, bu yerda

$$K_1(t,z) = \frac{1}{\sqrt{\pi}} \int_z^1 H_\eta(\eta,t) (\eta - z)^{-1/2} d\eta,$$

$$\tilde{\Phi}_1(t) = \int_0^1 \Phi_1(z) K_1(t,z) dz + [\varphi_3(0) - \varphi_1(0)]t + \varphi_1(0).$$

$K_1(t,z)$ va $\tilde{\Phi}_1(t)$ funksiyalarni tekshiramiz.

Avval $K_1(t,z)$ funksiyani qaraymiz. $t < z$ bo'lsa, u holda $t < \eta$ bo'ladi. Unda $H_\eta(t,\eta) = t$ bo'lib, ushbu

$$K_1(t,z) = \frac{t}{\sqrt{\pi}} \int_z^1 (\eta - z)^{-1/2} d\eta = \frac{2t}{\sqrt{\pi}} (1 - z)^{1/2}$$

tenglik o'rinli bo'ladi.

Agar $t \leq z$ bo'lsa,

$$K_1(t,z) = \frac{1}{\sqrt{\pi}} \int_z^t H_\eta(t,\eta) (\eta - z)^{-1/2} d\eta + \frac{1}{\sqrt{\pi}} \int_t^1 H_\eta(t,\eta) (\eta - z)^{-1/2} d\eta =$$

$$\frac{1}{\sqrt{\pi}} \int_z^t (t - 1) (\eta - z)^{-1/2} d\eta + \frac{1}{\sqrt{\pi}} \int_t^1 t (\eta - z)^{-1/2} d\eta = \frac{2t}{\sqrt{\pi}} (1 - z)^{1/2} - \frac{2}{\sqrt{\pi}} (t - z)^{1/2}.$$

Demak,

$$K_1(t, z) = \begin{cases} \frac{2t}{\sqrt{\pi}}(1-z)^{1/2}, & t < z, \\ \frac{2t}{\sqrt{\pi}}(1-z)^{1/2} - \frac{2}{\sqrt{\pi}}(t-z)^{1/2}, & t \geq z. \end{cases}$$

Bundan kelib chiqadiki, $K_1(t, z) \in C[0 \leq t, z \leq 1]$.

Endi $\tilde{\Phi}_1(t)$ funksiyani qaraymiz. $K_1(t, z)$ funksiyaning tuzilishini e'tiborga olsak,

$$\tilde{\Phi}_1(t) = \varphi_1(0) + [\varphi_3(0) - \varphi_1(0)]t + \frac{2t}{\sqrt{\pi}} \int_0^1 \Phi_1(z)(1-z)^{1/2} dz - \frac{2}{\sqrt{\pi}} \int_0^t \Phi_1(z)(t-z)^{1/2} dz.$$

Unda $\tilde{\Phi}_1'(t) = \varphi_3(0) - \varphi_1(0) + \frac{2}{\sqrt{\pi}} \int_0^1 \Phi_1(z)(1-z)^{1/2} dz - \frac{1}{\sqrt{\pi}} \int_0^t \Phi_1(z)(t-z)^{-1/2} dz,$

$$\tilde{\Phi}_1''(t) = -\frac{1}{\sqrt{\pi}} \frac{d}{dt} \left[t^{1/2} \int_0^1 \Phi_1(ts)(1-s)^{-1/2} ds \right].$$

Bu tengliklardan va $\Phi_1(t)$ funksiyaning xossaligidan foydalanib, ko'rsatish mumkinki, $\tilde{\Phi}_1(t) \in C^1[0,1] \cap C^2(0,1)$.

Demak, (23) - $\tau_2(t)$ funksiyaga nisbatan Fredgolmning ikkinchi tur integral tenglamasi ekan. Bu integral tenglama {(17),(20)} masalaga ekvivalent bo'lib, uning yechimining mavjudligi {(17),(20)} masala yechimining yagonaligidan kelib chiqadi va bu yechim $C^1[0,1] \cap C^2(0,1)$ sinfga qarashli bo'ladi. (23) tenglamadan $\tau_2(t)$ topilgandan so'ng, $\tau_3(t) = -\tau_2''(t)$ funksiya ham topilgan bo'ladi. Masala to'la hal etildi.

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