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VAQT YO'NALISHLARI TURLICHA BO'LGAN PARABOLO-GIPERBOLIK TENGLAMA UCHUN CHEGARAVIY MASALA

КРАЕВАЯ ЗАДАЧА ДЛЯ УРАВНЕНИЙ ПАРАБОЛО-ГИПЕРБОЛИЧЕСКОГО ТИПА С РАЗНЫМИ НАПРАВЛЕНИЯМИ ВРЕМЕНИ

BOUNDARY PROBLEM FOR A PARABOLIC-HYPERBOLIC EQUATIONS WITH DIFFERENT TIME DIRECTIONS

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Annotatsiya

Mazkur ishda vaqt yo'nalishlari turlicha bo'lgan aralash tipdagi bir tenglama uchun qo'yilgan chegaraviy masala yechimining mavjudligi va yagonalini isbotlangan.

Аннотация

В настоящей работе сформулирована краевая задача для уравнения параболо-гиперболического типа с различными направлениями времени и доказано существование и единственность задачи.

Abstract

In the present work, a boundary-value problem has been formulated for a parabolic-hyperbolic equation with different time directions and the existence and uniqueness of the problem has been proved.

Kalit so'zlar: *Aralash tipdagi tenglama, chegaraviy masala, integral tenglamalar usuli.*
Ключевые слова: *Уравнение смешанного типа, краевая задача, метод интегральных уравнений.*
Key words: *Equation of mixed type, boundary value problem, method of integral equations.*

KIRISH

xOy tekisligining $D = D_0 \cup D_1 \cup D_2$ sohasida ushbu

$$0 = Lu \equiv \begin{cases} u_{xx} - u_t = 0, & (x,t) \in D_0, \\ u_{xx} - u_{tt} = 0, & (x,t) \in D_1, \\ u_{tt} + u_x = 0, & (x,t) \in D_2 \end{cases} \quad (1)$$

tenglamani qaraylik, bu yerda $D_0 = \{(x,t) : 0 < x < +\infty,\, 0 < t < 1\}$, $D_1 = \{(x,t) : -x < t < x - 1,\, 0 < x < 1/2\}$, $D_2 = \{(x,t) : -1 < x < 0,\, 0 < t < 1\}$.

Ushbu $I_1 = \{(x,t) : -1 < x < 0,\ t = 0\}$, $I_2 = \{(x,t) : 0 < t < 1,\ x = 0\}$ belgilashlarni kiritib, (1) tenglama uchun D sohada quyidagi masalani o'rganamiz:

1-masala. $Lu = 0$ tenglamaning

$$u(x,0) = \varphi_1(x),\ 1 \le x < +\infty; \quad (2)$$

$$u(x,0) = \varphi_2(x),\ -1 \le x \le 0; \quad (3)$$

$$u(x,1) = \varphi_3(x),\ -1 \le x \le 0; \quad (4)$$

$$\lim_{x \to +\infty} u(x,t) = 0, \quad 0 \le t \le 1; \quad (5)$$

$$u(x,-x)=\psi_1(x),\ \ 0\le x\le 1/2; \qquad (6)$$

$$u(x,x-1)=\psi_2(x),\ \ 1/2\le x\le 1 \qquad (7)$$

shartlarni qanoatlantiruvchi $u(x,y)\in C(\overline{D})\cap C^1(D_0\cup D_1\cup D_2\cup I_1)$ regulyar yechimi topilsin, bu yerda $\varphi_i(x),\ i=\overline{1,3}$ va $\psi_j(x),\ i=\overline{1,2}$ berilgan funksiyalar.

Masalaning tadqiqoti. Qo‘yilgan masala yechimining mavjudligi va yagonalini isbotlaymiz. Shu maqsadda, masala shartlariga asoslanib, quyidagi belgilashlarni kiritaylik:

$$u(x,-0)=u(x,+0)=\tau_1(x),\quad 0\le x\le 1; \qquad (8)$$

$$u(-0,t)=u(+0,t)=\tau_2(t),\quad 0\le t\le 1; \qquad (9)$$

$$\lim_{x\to+0}u_x(x,t)=\lim_{x\to-0}u_x(x,t)=\tau_3(t),\quad 0<t<1, \qquad (10)$$

bu yerda $\tau_1(x)$, $\tau_2(t)$, $\tau_3(t)$ - noma’lum funksiyalar.

U holda masala yechimini D_1 sohada $u_{xx}-u_{tt}=0$ tenglama uchun (6) va $u(x,+0)=\tau_1(x),\quad x\in[0;1]$ shartlarni qanoatlantiruvchi yechimi sifatida ushbu

$$u(x,t)=\tau_1(x+t)+\psi_1\left(\frac{m+x-t}{2}\right)-\psi_1\left(\frac{m+x+t}{2}\right) \qquad (11)$$

ko‘rinishda aniqlashimiz mumkin [1]. (11) yechimni (7) shartga bo‘ysundirib, noma’lum $\tau_1(x)$ funksiyani ushbu ko‘rinishda topamiz:

$$\tau_1(x)=\psi_1\left(\frac{x+1}{2}\right)+\psi_2\left(\frac{x+1}{2}\right)-\psi_1(1),\quad 0\le x\le 1. \qquad (12)$$

Endi masalani D_0 sohada qaraylik. Ma’lumki, $u_{xx}-u_t=0$ tenglamaning D_0 sohaning yopig‘ida aniqlangan, uzluksiz hamda (2), (5) va ushbu

$$u(x,0)=\tau_1(x),\qquad 0\le x\le 1; \qquad (12)$$

$$\lim_{x\to+0}u_x(x,t)=\tau_3(t),\qquad 0<t<1; \qquad (13)$$

shartlarni qanoatlantiruvchi yechimi ushbu

$$u(x,t)=\int_0^1\frac{\sqrt{x\xi}}{2t}I_{-1/2}\left(\frac{x\xi}{2t}\right)e^{-(x^2+\xi^2)/4t}\,\tau_1(\xi)d\xi+$$
$$+\int_1^{+\infty}\frac{\sqrt{x\xi}}{2t}I_{-1/2}\left(\frac{x\xi}{2t}\right)e^{-(x^2+\xi^2)/4t}\,\varphi_1(\xi)d\xi-\frac{1}{\sqrt{\pi}}\int_0^t\tau_3(\eta)(t-\eta)^{-1/2}e^{-x^2/4(t-\eta)}d\eta \qquad (14)$$

ko‘rinishda aniqlanadi [2], bu yerda $I_p(z)-$ mavhum argumentli Bessel funksiyasi bo‘lib [3,4], quyidagi ko‘rinishda aniqlanadi:

$$I_p(z)=\sum_{k=0}^{+\infty}\frac{(z/2)^{2k-1/2}}{k!\Gamma(k+1/2)}.$$

(14) tenglikda x ni nolga intiltirib va (9) belgilashni e’tiborga olib, noma’lum $\tau_2(t)$ va $\tau_3(t)$ funksiyalar orasidagi D_0 sohadan olingan quyidagi

$$\tau_2(t)=-\frac{1}{\sqrt{\pi}}\int_0^t\tau_3(\eta)(\eta-t)^{-1/2}d\eta+\Phi_1(t),\qquad 0\le t\le 1 \qquad (15)$$

munosabatga ega bo‘lamiz, bu yerda

$$\Phi_1(t)=(\pi t)^{-1/2}\left[\int_0^1 \tau_1(\xi)e^{-\xi^2/4t}\,d\xi+\int_1^{+\infty}\varphi_1(\xi)e^{-\xi^2/4t}\,d\xi\right].$$

Endi masala shartlarini e'tiborga olib, $u_{xx}+u_t=0$ tenglama va (2), (3) shartlarda x ni nolga intiltiramiz. Natijada $\tau_2(t)$ va $\tau_3(t)$ noma'lumlar orasida ushbu

$$\tau_2''(t)+\tau_3(t)=0,\qquad 0<t<1 \tag{16}$$

ko'rinishdagi ikkinchi munosabatga va

$$\tau_2(0)=\varphi_1(0),\qquad \tau_2(1)=\varphi_3(0), \tag{17}$$

shartlarga ega bo'lamiz.

Agar (15), (16) va (17) munosabatlardan foydalanib, $\tau_2(t)$ va $\tau_3(t)$ noma'lum funksiyalarni bir qiymatli topsak, u holda D_0 sohada masalaning yechimi (14) formula orqali, D_2 sohada esa $u_{xx}+u_t=0$ tenglama uchun birinchi chegaraviy masalaning yechimi sifatida [4] ushbu

$$u(x,t)=\int_0^1 \tau_2(\eta)G_1(x,t;0,\eta)\,d\eta+\int_x^0 \varphi_2(\xi)G_{1\eta}(x,t;\xi,0)\,d\xi-\int_x^0 \varphi_3(\xi)G_{1\eta}(x,t;\xi,1)\,d\xi \tag{18}$$

ko'rinishda aniqlanadi[5,7], bu yerda

$$G_1(x,t;\xi,\eta)=\frac{1}{\sqrt{\pi(\xi-x)}}\sum_{n=-\infty}^{\infty}\left\{\exp\left[-\frac{(t-\eta-2n)^2}{4(\xi-x)}\right]-\exp\left[-\frac{(t+\eta-2n)^2}{4(\xi-x)}\right]\right\}.$$

Shuning uchun bundan buyon { (15), (16), (17)} tenglamalar sistemasini yechish bilan shug'ullanamiz. Kasr tartibli integral operator $D_{0t}^{-\alpha}$ ning ko'rinishidan foydalanib, (15) tenglikni quyidagi

$$\tau_2(t)=-D_{0t}^{-1/2}\tau_3(t)+\Phi_1(t),\qquad 0\le t\le 1$$

ko'rinishida yozib olamiz. Bu tenglikka $D_{0t}^{1/2}$ differensial operatorni tatbiq qilib va $D_{0t}^{-1/2}D_{0t}^{1/2}\tau_3(t)=\tau_3(t)$ tenglikni hisobga olib[ ], ushbu

$$\tau_3(t)=-D_{0t}^{1/2}\tau_2(t)+D_{0t}^{1/2}\Phi_1(t),\qquad 0<t<1 \tag{19}$$

tenglikka ega bo'lamiz. Buni (14) tenglamaga qo'yib, quyidagi

$$\tau_2''(t)-D_{0t}^{1/2}\tau_2(t)=-D_{0t}^{1/2}\Phi_1(t),\qquad 0<t<1 \tag{20}$$

integro-differensial tenglamani hosil qilamiz.

(20) tenglamaning (17) shartlarni qanoatlantiruvchi yechimi mavjudligini va yagonaligini isbotlaylik. Avval bir jinsli masalani, ya'ni

$$\tau_2''(t)-D_{0t}^{1/2}\tau_2(t)=0,\qquad 0<t<1; \tag{21}$$

$$\tau_2(0)=\tau_2(1)=0 \tag{22}$$

masalani qaraymiz. Faraz qilaylik, bu masala $\tau_2(t)\neq 0,\ 0\le t\le 1$ yechimga ega bo'lsin. Unda $\sup\left|\tau_2(t)\right|=\left|\tau_2(\xi)\right|\neq 0,\ \xi=const\in[0;1]$ bo'ladi. (22) shartlarga asosan $\xi\in(0,1)$. U holda $\tau_2(t)$ funksiya $t=\xi$ nuqtada musbat maksimumga yoki manfiy minimumga erishadi. Buni hamda butun hosilalarning xossalarini va kasr tartibli differensial operator uchun ekstremum prinsipini e'tiborga olsak [8], $\tau_2(\xi)-$musbat maksimum (manfiy minimum) bo'lganda

$$\tau_2''(\xi)\le 0(\ge 0),\qquad D_{0t}^{1/2}\tau_2(t)\Big|_{t=\xi}>0\ (<0)$$

tengsizliklar o'rinli bo'ladi. Bularga ko'ra

$$\tau_2''(\xi) - D_{0t}^{1/2}\,\tau_2(t)\Big|_{t=\xi} > 0\ (<0)$$

tengsizlik ham o‘rinli bo‘ladi. Bu tengsizlik (21) tenglamaga ziddir. Hosil bo‘lgan bu qarama-qarshilik $\tau_2(t) \neq 0$, $0 \le t \le 1$ degan farazimiz noto‘g‘ri ekanligini ko‘rsatadi. Demak, {(21), (22)} masala faqat trivial yechimga ega ekan. Bundan agar {(17),(20)} masalaning yechimi mavjud bo‘lsa, uning yagona bo‘lishi kelib chiqadi.

Endi {(17),(20)} masala yechimining mavjudligini ko‘rsatishga o‘tamiz. (20) tenglikni

$$\tau_2''(t) = D_{0t}^{1/2}\,\tau_2(t) - D_{0t}^{1/2}\,\Phi_1(t), \qquad 0 < t < 1$$

ko‘rinishida yozib, uning o‘ng tomonini ma’lum funksiya deb hisoblasak, {(17),(20)} masalaning yechimini

$$\tau_2(t) = \int_0^1 H(t,\eta)\, D_{0\eta}^{1/2}\left[\tau_2(\eta) - \Phi_1(\eta)\right]d\eta + [\varphi_3(0) - \varphi_1(0)]t + \varphi_1(0), \qquad t \in [0,1]$$

ko‘rinishida yozish mumkin [6,8], bu yerda

$$H(t,\eta) = \begin{cases} t(\eta - 1), & 0 \le t \le \eta; \\ \eta(t-1) & \eta \le t \le 1. \end{cases}$$

Bu tenglikda $\varphi_2(0) = 0$ ekanligini hisobga olib va $D_{0\eta}^{1/2} f(\eta)$ ifoda o‘rniga uning

$$D_{0\eta}^{1/2} f(\eta) \equiv \frac{1}{\sqrt{\pi}}\frac{d}{d\eta}\int_0^\eta (\eta - z)^{-1/2} f(z)\,dz$$

yoyilmasini qo‘yib, so‘ngra hosil bo‘lgan integralni bo‘laklab,

$$\tau_2(t) = \tilde{\Phi}_1(t) - \int_0^1 \tau_2(z) K_1(t,z)\,dz, \qquad 0 \le t \le 1 \qquad (23)$$

ko‘rinishdagi integral tenglamaga ega bo‘lamiz, bu yerda

$$K_1(t,z) = \frac{1}{\sqrt{\pi}}\int_z^1 H_\eta(\eta,t)(\eta - z)^{-1/2}d\eta,$$

$$\tilde{\Phi}_1(t) = \int_0^1 \Phi_1(z) K_1(t,z)dz + [\varphi_3(0) - \varphi_1(0)]t + \varphi_1(0).$$

$K_1(t,z)$ va $\tilde{\Phi}_1(t)$ funksiyalarni tekshiramiz.

Avval $K_1(t,z)$ funksiyani qaraymiz. $t < z$ bo‘lsa, u holda $t < \eta$ bo‘ladi. Unda $H_\eta(t,\eta) = t$ bo‘lib, ushbu

$$K_1(t,z) = \frac{t}{\sqrt{\pi}}\int_z^1 (\eta - z)^{-1/2}d\eta = \frac{2t}{\sqrt{\pi}}(1-z)^{1/2}$$

tenglik o‘rinli bo‘ladi.

Agar $t \le z$ bo‘lsa,

$$K_1(t,z) = \frac{1}{\sqrt{\pi}}\int_z^t H_\eta(t,\eta)(\eta - z)^{-1/2}d\eta + \frac{1}{\sqrt{\pi}}\int_t^1 H_\eta(t,\eta)(\eta - z)^{-1/2}d\eta =$$

$$\frac{1}{\sqrt{\pi}}\int_z^t (t-1)(\eta - z)^{-1/2}d\eta + \frac{1}{\sqrt{\pi}}\int_t^1 t(\eta - z)^{-1/2}d\eta = \frac{2t}{\sqrt{\pi}}(1-z)^{1/2} - \frac{2}{\sqrt{\pi}}(t-z)^{1/2}.$$

Demak,

$$K_1(t,z)=\begin{cases}\dfrac{2t}{\sqrt{\pi}}(1-z)^{1/2}, & t<z,\\ \dfrac{2t}{\sqrt{\pi}}(1-z)^{1/2}-\dfrac{2}{\sqrt{\pi}}(t-z)^{1/2}, & t\geq z.\end{cases}$$

Bundan kelib chiqadiki, $K_1(t,z)\in C\left[0\leq t,\ z\leq 1\right]$.

Endi $\tilde{\Phi}_1(t)$ funksiyani qaraymiz. $K_1(t,z)$ funksiyaning tuzilishini e'tiborga olsak,

$$\tilde{\Phi}_1(t)=\varphi_1(0)+[\varphi_3(0)-\varphi_1(0)]t+\frac{2t}{\sqrt{\pi}}\int_0^1\Phi_1(z)(1-z)^{1/2}dz-\frac{2}{\sqrt{\pi}}\int_0^t\Phi_1(z)(t-z)^{1/2}dz.$$

Unda $\tilde{\Phi}_1'(t)=\varphi_3(0)-\varphi_1(0)+\dfrac{2}{\sqrt{\pi}}\displaystyle\int_0^1\Phi_1(z)(1-z)^{1/2}dz-\dfrac{1}{\sqrt{\pi}}\displaystyle\int_0^t\Phi_1(z)(t-z)^{-1/2}dz$,

$$\tilde{\Phi}_1''(t)=-\frac{1}{\sqrt{\pi}}\frac{d}{dt}\left[t^{1/2}\int_0^1\Phi_1(ts)(1-s)^{-1/2}ds\right].$$

Bu tengliklardan va $\Phi_1(t)$ funksiyaning xossalaridan foydalanib, ko'rsatish mumkinki, $\tilde{\Phi}_1(t)\in C^1\left[0,1\right]\cap C^2(0,1)$.

Demak, (23) - $\tau_2(t)$ funksiyaga nisbatan Fredgolmning ikkinchi tur integral tenglamasi ekan. Bu integral tenglama {(17),(20)} masalaga ekvivalent bo'lib, uning yechimining mavjudligi {(17),(20)} masala yechimining yagonaligidan kelib chiqadi va bu yechim $C^1\left[0,1\right]\cap C^2(0,1)$ sinfga qarashli bo'ladi. (23) tenglamadan $\tau_2(t)$ topilgandan so'ng, $\tau_3(t)=-\tau_2''(t)$ funksiya ham topilgan bo'ladi. Masala to'la hal etildi.

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