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ИҚТИСОДИЁТ

ТАРИХ

УДК: 517.93

DIFFERENTIAL GAMES OF THE SECOND ORDER
ДИФФЕРЕНЦИАЛЬНЫЕ ИГРЫ ВТОРОГО ПОРЯДКА
ИККИНЧИ ТАРТИБЛИ ДИФФЕРЕНЦИАЛ ЎЙИНЛАР

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Annotation

In this paper, we study the pursuit-evasion problem for the second order differential game when the initial positions of moving objects are linearly dependent and controls of the players have geometric constraints. The newsufficient solvability conditions are obtained for problems of the pursuit and evasion.

Аннотация

В настоящей работе изучается задача преследования-убегания для дифференциальных игр второго порядка, когда начальные состояния и начальные скорости игроков линейно зависимы при геометрических ограничениях на управления. Получены новые достаточные условия разрешимости для задач преследования и убегания.

Аннотация

Мақолада ҳаракатланувчи объектларнинг бошланғич ҳолатлари ва бошланғич тезликлари чизиқли боғлиқ ҳамда бошқарувлари геометрик чегараланишга эга ҳол учун иккинчи тартибли дифференциал ўйинларда қувиш-қочиш масаласи ўрганилган. Бунда қувувчи ва қочувчи учун янги етарлилик шартлари таклиф қилинган.

Keywords and expressions: differential game, acceleration, geometric constraint, evader, pursuer, initial positions, strategy.

Ключевые слова и выражения: дифференциальная игра, ускорение, геометрическое ограничение, убегающий, преследователь, начальные состояния, стратегия.

Таянч сўз ва иборалар: дифференциал ўйин, тезланиш, геометрик чегараланиш, қочувчи, қувловчи, бошланғич ҳолат, стратегия.

I. Introduction

Early sample of the Pursuit-Evasion problems is generally assumed to begin with a problem posed and solved in 1732 by the French mathematician and hydrographer Pierre Bouguer [22]. A more recent treatment of the history appear in the book P.Nahin [22]. But Pursuit-Evasion of the problems began to be studied systematically by the American mathematician Rufus Isaacs in 50's. The concept of" Differential Games" first appeared in his series of secret works of the project of Corporation RAND (USA). R.Isaacs studies were published in the form of monographs [13], which contained a great deal of brilliant differential game examples. The Author looked at them as problems of Variation Calculus and tried to apply the Hamilton-Jacoby method now known as Isaacs' method. But the subject turned out a far complicated for classical methods. The idea used by R.Isaacs had heuristic character only. Never the less the book [13] created interest to new problems. It was then that mathematicians and mechanics, specialists and amateurs began to consider differential games.

Modern Differential Games set the theory of development of mathematical methods of control processes, combines the dynamism, control, fighting, awareness, and optimal number of other important qualities, and represent one of the most complicated mathematical models of real processes having great practical importance. The Theory's foundation was settled by the mathematicians W.Fleming [9], A.Friedman [10], O.Hajek [11-12], L.S.Pontryagin [29], N.N.Krasovskiy [18-19], L.A.Petrosyan [23-28], B.N.Pshenichnyi [30]. This authors settled their own approach to the subject. Its further development was achieved by many specialists [1-7, 8-9, 14-17, 20, 21, 31-41 and others].

At the present time there are more than a hundred monographs on the theory. Never the less completely solved samples of Differential Games are quite few. In this paper, we study the pursuit-evasion problem for the second order differential game when the initial positions of moving objects are linearly dependent and controls of the players have geometric constraints. The new sufficient solvability conditions are obtained for problems of the pursuit and evasion.

II. Formulation of the problems

Let **P** and **E** objects with opposite aim be given in the space $\boldsymbol{R}^n$ and their movements based on the following differential equations and initial conditions

P: $\ddot{x} = u\,,\quad x_1 - kx_0 = 0\,,\quad |u| \le \alpha\,,$ (1)

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E $: \ddot{y} = v, y_1 - ky_0 = 0, |v| \le \beta$, (2)

where $x, y, u, v \in \boldsymbol{R^n}$; x – a position of **P** object inthe space $\boldsymbol{R^n}$, $x_0 = x(0), \quad x_1 = \dot{x}(0)$ – its initial position and velocity respectively at $t = 0$; u – being a controlled acceleration of the pursuer, mapping $u : [0, \infty) \to \boldsymbol{R^n}$ and it is chosen as a measurable function with respect to time; we denote a set of all measurable functions $u(\cdot)$ that satisfies the condition $|u| \le \alpha$ by $\boldsymbol{U}$. y – a position of **E** object in the space $\boldsymbol{R^n}$, $y_0 = y(0), \; y_1 = \dot{y}(0)$ – its initial position and velocity respectively at $t = 0$; v – being a controlled acceleration of the evader, mapping $v : [0, \infty) \to \boldsymbol{R^n}$ and it is chosen as a measurable function with respect to time; we denote a set of all measurable functions $v(\cdot)$ that satisfies the condition $|v| \le \beta$ by $\boldsymbol{V}$.

Definition 1. For a trio of $(x_0, x_1, u(\cdot)), u(\cdot) \in \boldsymbol{U}$, the solution of the equation (1), that is, $x(t) = x_0 + x_1 t + \int_0^t \int_0^s u(\tau) d\tau ds$ is called a trajectory of the pursuer on interval $t \ge 0$.

Definition 2. For a trio of $(y_0, y_1, v(\cdot)), \; v(\cdot) \in \boldsymbol{V}$, the solution of the equation (2), that is, $y(t) = y_0 + y_1 t + \int_0^t \int_0^s v(\tau) d\tau ds$ is called a trajectory of the evader on interval $t \ge 0$.

Definition 3. The pursuit problem for the differential game (1)−(2) is called to be solved if there exists such control function $u^*(\cdot) \in \boldsymbol{U}$ of the pursuer for any control function $v(\cdot) \in \boldsymbol{V}$ of the evader and the following equality is carried out at some finite time t^*

$$x(t^*) = y(t^*). \tag{3}$$

Definition 4. For the problem (1)−(2), time T is called a guaranteed pursuit time if it is equal to an upper boundary of all the finite values of pursuit time t^* satisfying the equality (3).

Definition 5. For differential game (1)−(2), an evasion problem is said to be held however, the pursuer chooses any control function $\forall u(\cdot) \in \boldsymbol{U}$, if there exists $\exists v^*(\cdot) \in \boldsymbol{V}$ for the evader and the following condition is true for the trajectories $x(t)$, $y(t)$ that is found according to those control functions:

$$x(t) \ne y(t), \quad t \ge 0 \tag{4}$$

III. The solve of the pursuit problem

Definition 6. For the differential game (1)−(2), the following function is called П-strategy of the pursuer ([2]-[4]):

$$u(v) = v - \lambda(v)\xi_0, \tag{5}$$

where $\lambda(v, \xi_0) = (v, \xi_0) + \sqrt{(v, \xi_0)^2 + \alpha^2 - |v|^2}$, $\xi_0 = z_0 / |z_0|$, (v, ξ_0) is the scalar product of thevectors v and ξ_0 in the space $\boldsymbol{R^n}$.

Property 1. If $\alpha \ge \beta$, then a function $\lambda(v, \xi_0)$ is continuous, nonnegative and defined for all v that satisfies the inequality $|v| \le \beta$.

Property 2. If $\alpha \ge \beta$, then the following inequality is true for the function $\lambda(v, \xi_0)$:

$$\alpha - |v| \le \lambda(v, \xi_0) \le \alpha + |v|.$$

Theorem 1. If one of the following conditions holds for the second order differential game (1)−(2), that is, 1. $\alpha = \beta$ and $k < 0$; or 2. $\alpha > \beta$ and $k \in R$, then by virtue of strategy (5) the guaranteed pursuit time becomes as follows

$$T = \begin{cases} \left(|z_0|k + \sqrt{|z_0|^2 k^2 + 2|z_0|(\alpha-\beta)}\right)/(\alpha-\beta), & \text{if } k \neq 0 \text{ and } \alpha > \beta, \\ -1/k, & \text{if } k < 0 \text{ and } \alpha = \beta, \\ \sqrt{2|z_0|/(\alpha-\beta)}\ , & \text{if } k = 0 \text{ and } \alpha > \beta. \end{cases}$$

Proof. Suppose the pursuer choose the strategy in the form (5) when the evader chooses any control function $v(\cdot) \in V$. Then, according to the equations (1)−(2), we have the following Caratheodory's equation:

$$\ddot{z} = -\lambda\big(v(t)\big)\xi_0, \quad \dot{z}(0) - kz(0) = 0$$

Thus the following solution will be found by the given initial conditions

$$z(t) = z_0(kt+1) - \xi_0 \int_0^t\int_0^s \lambda\big(v(\tau),\xi_0\big)d\tau ds$$

or

$$|z(t)| = |z_0|(kt+1) - \int_0^t\int_0^s \left(\big(v(\tau),\xi_0\big) + \sqrt{(v(\tau),\xi_0)^2 + \alpha^2 - |v(\tau)|^2} \right) d\tau ds .$$

According to the properties 1−2, we will form the following inequalities

$$|z(t)| \le |z_0|(kt+1) - \int_0^t\int_0^s (\alpha - |v(\tau)|) d\tau ds,$$

$$|z(t)| \le |z_0|(kt+1) + t^2(\beta-\alpha)/2 .$$

We denote

$$f\big(t,a,k,\alpha,\beta\big) = a\big(kt+1\big) - \frac{t^2}{2}\big(\alpha-\beta\big),\ a = |z_0|. \tag{6}$$

1. Let be $\alpha = \beta$.

1.1. If $k > 0$, then $f\big(t,a,k,\alpha,\beta\big) = a\big(kt+1\big)$ and it is increasing function (Fig.1).

1.2. If $k = 0$, then $f\big(t,a,k,\alpha,\beta\big) = |z_0|$ is constant function (Fig.2).

1.3. If $k < 0$, then the function (6) is decreasing and it equals to zero at the time $t^* = -1/k$ (Fig.3)

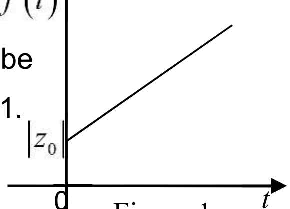


Figure 1.

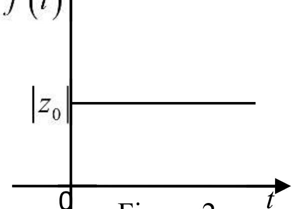


Figure 2.

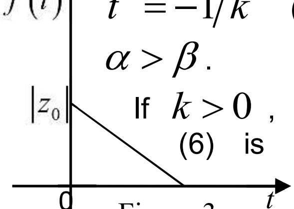


Figure 3.

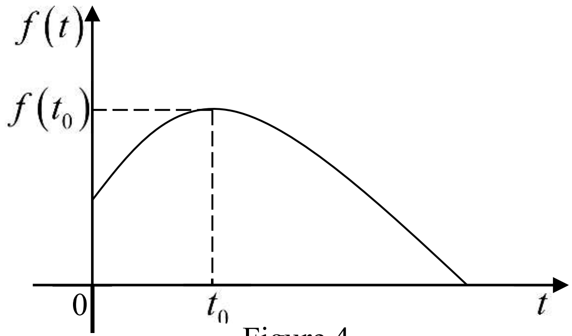
2. Let be $\alpha > \beta$.

2.1. If $k > 0$, then the function (6) is equal to zero at the time

$T_{>} = \left(|z_0| k + \sqrt{|z_0|^2 k^2 + 2|z_0|(\alpha - \beta)} \right) / (\alpha - \beta)$ (Fig.4).

Maximal value of the function (6) is $f(t_0) = \left(2|z_0|(\alpha - \beta) + |z_0|^2 k^2 \right) / 2(\alpha - \beta)$ at moment $t_0 = |z_0| k / (\alpha - \beta)$.

2.2. If $k < 0$, then the function (6) decreases monotonically and this function turns to zero at time $T_{<}$ as in the case 2.1 (Fig.5).



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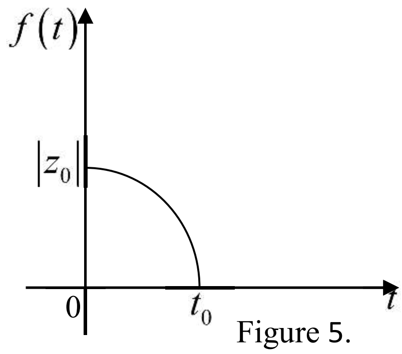

Figure 5.

2.3. If $k = 0$, then $f(t, a, k, \alpha, \beta) = a - \frac{t^2}{2}(\alpha - \beta)$ and the pursuit time equals, to $T_0 = \sqrt{2|z_0|/(\alpha - \beta)}$.

In conclusion, the relation (3) is true at some time t^* according to the inequality $|z(t)| \le |z_0|(kt + 1) + t^2(\beta - \alpha)/2$ and properties of (6), and it is determined that a relation $t^* \le T$ is correct, i.e., the pursuit problem is solved, which completes the proof of the Theorem 1.

IV. The solve of the evasion problem

To solve the evasion problem we will propose a strategy of the evader as follows:

Definition 7. In differential game (1) – (2) we call the strategy of the evader the following function:

$$v^*(t) = -\frac{z_0}{|z_0|}\beta, \quad t \ge 0, \tag{7}$$

where $z_0 = x_0 - y_0 \ne 0$.

Theorem 2. If one of the following conditions holds:

1. $\alpha = \beta$ and $k \ge 0$; or 2. $\alpha < \beta$ and $k \in \left(-\sqrt{2(\alpha - \beta)/|z_0|};\ +\infty \right)$,

then for differential game (1)–(2), the evasion problem is solved by the strategy of the evader (7) and a change function between the objects will be in the following form:

$$f(t, k, \alpha, \beta, |z_0|) = \begin{cases} |z_0| kt + |z_0|, & \alpha = \beta, \\ \frac{\beta - \alpha}{2} t^2 + |z_0| kt + |z_0|, & \alpha < \beta. \end{cases}$$

Proof. Suppose, let the pursuer choose any control function $u(\cdot)\in \boldsymbol{U}$ and the evader choose the control function (7). Then according to (1)–(2) we have the following solutions:

$$x(t)=x_0+tx_1+\int_0^t\int_0^s u(\tau)d\tau ds,$$

$$y(t)=y_0+ty_1+\int_0^t\int_0^s v^*(\tau)d\tau ds.$$

Now write their distinction function:

$$z(t)=x(t)-y(t)=z_0+z_1t+\int_0^t\int_0^s u(\tau)d\tau ds+\int_0^t\int_0^s \frac{z_0}{|z_0|}\beta d\tau ds,$$

where $z_1=\dot{x}(0)-\dot{y}(0)$. If we subtract the initial conditions, we form a relation $z_1=kz_0$. From this, we have the following equality:

$$z(t)=z_0(kt+1)+\frac{z_0}{|z_0|}\beta\frac{t^2}{2}+\int_0^t\int_0^s u(\tau)d\tau ds.$$

Evaluate the absolute value of this function from low:

$$|z(t)|\geq\left|z_0(kt+1)+\frac{z_0}{|z_0|}\beta\frac{t^2}{2}\right|-\left|\int_0^t\int_0^s u(\tau)d\tau ds\right|\geq$$

$$\geq|z_0|(kt+1)+\beta\frac{t^2}{2})-\int_0^t\int_0^s|u(\tau)|d\tau ds\geq|z_0|(kt+1)+(\beta-\alpha)\frac{t^2}{2}.$$

We will consider as a parametric function the right side of the latest inequality:

$$f(k,t,\alpha,\beta,|z_0|)=|z_0|(kt+1)+\frac{\beta-\alpha}{2}t^2. \quad (8)$$

To check properties of the function (8) we will introduce some simplifications, i.e., $|z_0|=a$, $\frac{\beta-\alpha}{2}=\gamma$. Therefore, the function (8) becomes in the following form:

$$f(k,t,\gamma)=a+akt+\gamma t^2. \quad (9)$$

Now we will check the function (9) with respect to parameters ρ,σ and k.

1. Let be $\alpha=\beta$. Then $f(k,t)=a+akt$ and we will analyze this function in respect of a sign of parameter k:

1.1. Let be $k>0$. Consequently $ak>0$, and there doesn't exist a positive solution t which the function equals to zero. So in this case, the evasion holds (Fig.6).

1.2. Let be $k=0$. Thus $f(k,t)=a$, and in this case, a distance between the pursuer and evader don't change. So the evasion holds (Fig.7).

1.3. Let be $k<0$. Then $ak<0$. The function $f(t,k)$ has a positive solution $t=-1/k$. So the evasion doesn't hold (Fig.8).

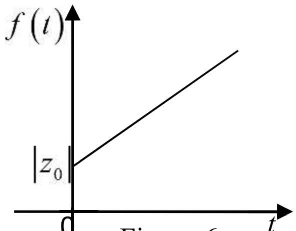

Figure 6.

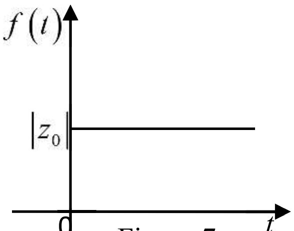

Figure 7.

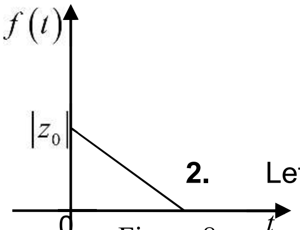

Figure 8.

2. Let be $\alpha < \beta$. Then $\gamma > 0$ in the function (9). We will analyze the function (9) in relation to a sign of parameter k.

2.1. Let be $k < 0$. Then $t_0 = -\dfrac{ak}{2\gamma} > 0$ is a minimum approach point. In order to being the evasion held a discriminant $D = a^2k^2 - 4a\gamma$ must be negative, i.e., $D = a^2k^2 - 4a\gamma < 0$. Therefore, $k^2 < \dfrac{4\gamma}{a}$ and we have an interval $k \in \left(-\sqrt{\dfrac{2(\beta-\alpha)}{|z_0|}}, \sqrt{\dfrac{2(\beta-\alpha)}{|z_0|}} \right)$. If we unite the latest interval with an interval $k < 0$, then the evasion holds on interval $k \in \left(-\sqrt{\dfrac{2(\beta-\alpha)}{|z_9|}}, 0 \right)$ (Fig.9).

2.2. Let be $k = 0$. Then $f(t,\gamma) = a + \gamma t^2$ and there doesn't exist a pursuit time because of $\gamma > 0$. Thus the evasion holds (Fig.10).

2.3. Let be $k > 0$. Consequently, in the function (9), $t_0 < 0$. Hence there is no a positive solution t which the function equals to zero. So in this case, the evasion holds (Fig.11).

In conclusion, the relation (4) is true in all values of interval $t \geq 0$ according to the inequality $|z(t)| \geq f\left(k, t, \alpha, \beta, |z_0|\right)$ and properties of (9), that is, the evasion problem is solved, which completes the proof of the Theorem 2.

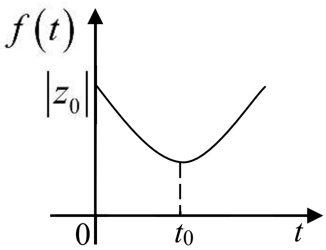

Figure 9.

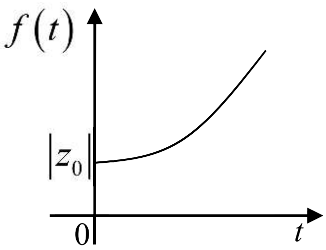

Figure 10.

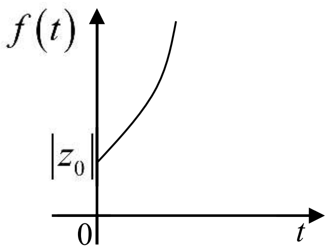

Figure 11.

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