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${}^{2}R_{5}$ ФАЗО ФАЗООСТЛАРИНИНГ ГЕОМЕТРИЯЛАРИ

О ГЕОМЕТРИИ НА ПОДПРОСТРАНСТВАХ В ${}^{2}R_{5}$

ABOUT GEOMETRY ON SUBSPACES IN ${}^{2}R_{5}$

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Annotatsiya

Ushbu maqolada besh o‘lchovli ikki indeksli psevdoevklid fazosining fazoosti geometriyalari va ko‘pxilliklari o‘rganilgan. Geometriya haqiqiy radiusli sferada aniqlanadi. Uch va to‘rt o‘lchovli barcha giperbolik fazolarning mavjudligi, shuningdek maksimal o‘lchamdagi ko‘pxilliklar isbotlangan.

Аннотация

В работе исследована геометрия и её многообразия на подпространствах, пятимерное псевдоевклидово пространство индекса два. Определена геометрия в сфере действительного радиуса. Доказано существование всех гиперболических пространств размерности трёх и некоторых четырехмерных гиперболических пространств, а также многообразие максимальной размерности.

Abstract

The paper investigates geometry and its manifolds on subspaces, a five-dimensional pseudo-Euclidean space of index two. Geometry in the sphere of real radius is defined. The existence of all hyperbolic spaces of dimension three and some four-dimensional hyperbolic spaces is proved, as well as a manifold of maximum dimension.

Kalit so’zlar: *Ko’pxillik, fazoosti, psevdoizotrop fazo, giperbolik fazo, ajraluvchi metrika, izotrop vektor, haqiqiy, mavhum va nol radiusli sferalar, izotrop konus.*

Ключевые слова: *Многообразие, подпространство, псевдоизотропные пространства, гиперболические пространства, вырожденная метрикой, изотропные векторы, сфера действительного, мнимого, нулевого радиуса, изотропный конус.*

Key words: *Manifold, subspace, pseudoisotropic spaces, hyperbolic spaces, degenerate metric, isotropic vectors, sphere of real, imaginary, zero radius, isotropic cone*

INTRODUCTION

In the geometrization of physical problems or problems of quantum mechanics, the method of dimension reduction is used. For this purpose, some of the coordinates are considered constant or linearly dependent on each other. Thus, the problem is not considered in the original space, but is considered in its subspace [1].

In Euclidean multidimensional geometry, reducing dimensionality does not affect the geometry of space. But in pseudo-Euclidean space, the reduction in dimensionality corresponding to these physical methods often changes the geometry of the space in question [2]-[3]. Therefore, in this we will prove the existence of various geometries in the subspaces of space ${}^{2}R_{5}$. Basically, we will consider three-dimensional and four-dimensional subspaces [5]-[6].

In physics, by equating some coordinates of the space under consideration, they achieve a reduction in the dimension of the space in which the experiment is carried out. When a space with a positive definite metric or a metric space is considered, this technique leads to a space of lower dimensionality, similar to the one in the space under consideration.

But in the geometry of pseudo-Euclidean space, a lower-dimensional space obtained by equating some coordinates will lead to a geometry quite different from the geometry in question.

Basic definitions

Let us consider a pseudo-Euclidean space of ${}^2R_5$ dimension five and index two [2].

If $O\{\vec{e}_1, \vec{e}_2, \vec{e}_3, \vec{e}_4, \vec{e}_5\}$ is an orthogonal and unit coordinate system, then the scalar product of vectors $\vec{X}(x_1, x_2, x_3, x_4, x_5)$ and $\vec{Y}(y_1, y_2, y_3, y_4, y_5)$ is determined by the formula

$$\left(\vec{X}\cdot\vec{Y}\right) = x_1y_1 + x_2y_2 + x_3y_3 - x_4y_4 - x_5y_5 \quad (1)$$

The vector norm is $\left|\vec{X}\right| = \sqrt{\left(\vec{X}\cdot\vec{X}\right)}$, and the distance between points $A(x_1, x_2, x_3, x_4, x_5)$ and $B(y_1, y_2, y_3, y_4, y_5)$ is defined as the vector norm $\overrightarrow{AB}$, and in coordinate form it is

$$\left|\overrightarrow{AB}\right| = \sqrt{\left(\overrightarrow{AB}\cdot\overrightarrow{AB}\right)} = \sqrt{(y_1-x_1)^2 + (y_2-x_2)^2 + (y_3-x_3)^2 - (y_4-x_4)^2 - (y_5-x_5)^2} \quad .$$

(2)

Obviously, the distance between points can take a real and imaginary value, and will also be isotropic if the distance is zero and the points do not coincide. The corresponding vectors are also called real, imaginary and isotropic vectors.

The set of isotropic vectors ${}^2R_5$ form an isotropic cone of space and the coordinates of the points of the isotropic cone satisfy the equation

$$x_1^2 + x_2^2 + x_3^2 - x_4^2 - x_5^2 = 0. \quad (3)$$

Isotropic cone (3) – limits the set of real norm vectors and imaginary vectors.

In pseudo-Euclidean space ${}^2R_5$ - there are three types of sphere, defined as a set of points equidistant from a given point, which is called the radius.

First $x_1^2 + x_2^2 + x_3^2 - x_4^2 - x_5^2 = r^2$ – sphere actual radius.

Second $x_1^2 + x_2^2 + x_3^2 - x_4^2 - x_5^2 = -r^2$ – sphere imaginary radius.

The third is a sphere of zero radius, which coincides with the isotropic cone (3) of space.

Note that the spheres of space ${}^2R_5$ are hyperbolic surfaces, since the three-dimensional and four-dimensional sections of these spheres are the Euclidean sphere or hyperbolic surfaces.

It is known [3] that the set of points on a sphere with identified diametrically opposite points of pseudo-Euclidean space is isometric to hyperbolic spaces, that is, Riemannian spaces of negative curvature.

Let us find out which hyperbolic spaces the geometry in the sphere of pseudo-Euclidean space ${}^2R_5$ is isometric to.

Theorem 1. A set with identified diametrically opposite points of a sphere of real radius of space ${}^2R_5$ is isometric to hyperbolic space ${}^2S_4$.

Proof: Diametrically opposite points of a sphere of real radius are symmetrical with respect to the origin. Therefore, the coordinates of these points differ in sign. Accepting condition $x_1 > 0$, we identify diametrically opposite points of the sphere of real radius.

Consider the set of M points on the sphere of space ${}^2R_5$. The coordinates of these points satisfy the condition

$$M:\begin{cases}\{x_1, x_2, x_3, x_4, x_5\}\\ x_1^2+x_2^2+x_3^2-x_4^2-x_5^2=r^2\\ x_1>0.\end{cases}$$

Let $A(x_1^{'}, x_2^{'}, x_3^{'}, x_4^{'}, x_5^{'})$ and $B(x_1^{''}, x_2^{''}, x_3^{''}, x_4^{''}, x_5^{''})$ points of set M and vectors $\overrightarrow{OA}=\vec{X}$, $\overrightarrow{OB}=\vec{Y}$ be the radius vectors of these points. Then, through the origin of coordinates $O(0, 0, 0, 0, 0)$ and points A and B, a two-dimensional plane of space ${}^2R_5$ can be drawn.

The section of this plane with a sphere of real radius is a circle of large diameter of the sphere of real radius. We consider the arc of this circle to be a straight line in the set M.

Then we define the length of the segment AB in the set M as proportional to the angle between the vectors $\overrightarrow{OA}$ and $\overrightarrow{OB}$ - the space ${}^2R_5$.

According to the formula for the angle between vectors in ${}^2R_5$, we have

$$ch\frac{\delta}{r}=\frac{(\vec{X}\cdot\vec{Y})}{|\vec{X}||\vec{Y}|}$$

where $(\vec{X}\cdot\vec{Y})$ is the scalar product and $|\vec{X}|, |\vec{Y}|$ is the norm of vectors in ${}^2R_5$.

Therefore, the distance δ between points A and B, set M is calculated by the formula

$$ch\frac{\delta}{r}=\frac{x_1y_1+x_2y_2+x_3y_3-x_4y_4-x_5y_5}{\sqrt{x_1^2+x_2^2+x_3^2-x_4^2-x_5^2}\cdot\sqrt{y_1^2+y_2^2+y_3^2-y_4^2-y_5^2}} \tag{4}$$

where the coordinates satisfy the condition of belonging to a sphere with a real radius of ${}^2R_5$.

If the coordinates $\{x_1, x_2,..., x_n\}$ are considered to be the projective coordinates of points in space, then the distance between the points of the set. M defined by formula (4), gives the hyperbolic space distance formula ${}^2S_4$. The theorem is proven.

For example, [4] it was proven that in subspace $M(x_1, x_2, x_3, x_2, x_3)\subset {}^2R_5$ the geometry of Galilean space is generated. It is easy to notice $M\subset {}^2R_5$, $x_2=x_4$ and $x_3=x_5$ in the subspace under consideration. The geometry in ${}^2R_5-$ is pseudo-Euclidean, and the geometry of its subspace $M(x_1, x_2, x_3, x_2, x_3)$ is Galilean. Galilean space has a degenerate metric, and this geometry is completely different from the geometry of pseudo-Euclidean space.

We will define the geometry of some subspaces ${}^2R_5$ that have a geometry different from the geometry of pseudo-Euclidean space [5]-[6].

Let us find out in which subspaces of space ${}^2R_5$ spaces with degenerate metrics appear.

Theorem 2. Subspace $I(x_1, x_2, x_3, x_3, x_4)\subset {}^2R_5$ is a pseudoisotropic space ${}^{10}R_4^3$.

Proof. Consider subspace $I(x_1, x_2, x_3, x_3, x_4)$. If points $A(x_1, x_2, x_3, x_3, x_4)$ and $B(y_1, y_2, y_3, y_3, y_4)$ are points $I(x_1, x_2, x_3, x_3, x_4)$, then the distance between these points, calculated using the formula $|AB|=\sqrt{(y_1-x_1)^2+(y_2-x_2)^2+(y_3-x_3)^2-(y_4-x_4)^2-(y_5-x_5)^2}$, has the form

$$d_{AB}=|AB|=\sqrt{(y_1-x_1)^2+(y_2-x_2)^2+(y_3-x_3)^2-(y_3-x_3)^2-(y_4-x_4)^2}$$

or

$$d_{AB}=|AB|=\sqrt{(y_1-x_1)^2+(y_2-x_2)^2-(y_4-x_4)^2}\ .$$

This is the metric of the three-dimensional Minkowski space ${}^1R_3$. From the equality to zero of this metric it does not follow that points A and B coincide. Between these points there is a segment equal to $|y_3-x_3|$. If we take it into account as the second distance between A and B, we obtain a degenerate metric of the pseudoisotropic space ${}^{10}R_4^3$. The theorem is proven.

The following corollaries follow from Theorem 2.

Corollary 1. Under condition $x_4=0$, the subspace $I_0(x_1, x_2, x_3, x_3, 0)\subset I(x_1, x_2, x_3, x_3, x_4)$ will be a three-dimensional isotropic space R_3^2.

Corollary 2. The set of points on the sphere of subspace ${}^{10}R_4^3$ is isometric to the semihyperbolic space ${}^{10}S_3^2$.

When $x_i=0,\ i=1, 2, 3$, a hyperbolic space ${}^2S_3$ appears at its section with the sphere.

Also, at $x_4=0$ or $x_5=0$, a hyperbolic space ${}^1S_3$ appears on the section, that is, three-dimensional Lobachevsky space.

Thus, we can conclude that subspaces ${}^2R_5$ contain all hyperbolic spaces of the third dimension. Moreover, they appear when the sphere of space ${}^2R_5$ intersects with hyperplanes.

But in sections of the sphere of space ${}^2R_5$ with subspaces with a degenerate metric, semi-hyperbolic spaces appear.

Let us prove that four-dimensional spaces are always pseudo-Euclidean or semi-Euclidean spaces.

Lemma. The hyperplane of pseudo-Euclidean space ${}^2R_5$ has a pseudo-Euclidean or semi-Euclidean metric.

Proof. Let α be a hyperplane of space ${}^2R_5$. Then its equation in affine coordinates has the form

$$a_1x_1+a_2x_2+a_3x_3+a_4x_4+a_5x_5=0,$$

with $\vec{N}(a_1, a_2, a_3, -a_4, -a_5)$ – being the normal vector.

Since α is a hyperplane, there are four linearly independent vectors in it.

In space ${}^2R_5$ there are five linearly independent vectors. Since the normal vector $\vec{N}$ does not belong to the plane α, it, together with four linearly independent vectors $\vec{\alpha}_1, \vec{\alpha}_2, \vec{\alpha}_3, \vec{\alpha}_4$ of this plane, forms the basis of the space ${}^2R_5$.

The normal vector $\vec{N}$ can be real, imaginary, or isotropic. Regardless of what vector $\vec{N}$ may be, among the α_i vectors, at least one must be imaginary. This means that the metric on the α plane will be pseudo-Euclidean. The lemma is proven.

Theorem 3. The maximum size of a variety M in ${}^2R_5$ can be equal to three.

Proof. It is easy to prove that there is a three-dimensional manifold in ${}^{2}R_{5}$. An example of a three-dimensional manifold is the hyperplane $x_4 = x_5 = 0$, since on this manifold the metric is positive definite, it is Euclidean.

From the lemma proved above, it follows that any tangent space of a fourth-order set has a pseudo-Euclidean metric. It follows that there is no four-dimensional subspace in ${}^{2}R_{5}$ that would be a manifold. The theorem is proven.

CONCLUSION

It has been established that the internal geometry of submanifolds in pseudo-Euclidean space does not always coincide with the geometry of the space itself. It turns out that in pseudo-Euclidean space there are subspaces that have a Euclidean metric, or a degenerate metric. This peculiarity of the metric of pseudo-Euclidean space was studied for two-dimensional planes and the following was proved.

The existence of a four-dimensional pseudoisotropic space ${}^{10}R_{4}^{3}$ has been proven. In four-dimensional subspaces ${}^{2}R_{5}$ there is no three-dimensional elliptic space S_3, but there is a subspace where the geometry will be semi-elliptic. Some three-dimensional spaces with projective metrics that exist in ${}^{2}R_{5}$ subspaces are presented.

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